9.1 - Introduction to Oscillatory Motion and Waves class="introduction"

There are at least four types of waves in this picture—only the water waves are evident. There are also sound waves, light waves, and waves on the guitar strings. (credit: John Norton)

What do an ocean buoy, a child in a swing, the cone inside a speaker, a guitar, atoms in a crystal, the motion of chest cavities, and the beating of hearts all have in common? They all **oscillate**—-that is, they move back and forth between two points. Many systems oscillate, and they have certain characteristics in common. All oscillations involve force and energy. You push a child in a swing to get the motion started. The energy of atoms vibrating in a crystal can be increased with heat. You put energy into a guitar string when you pluck it.

Some oscillations create **waves**. A guitar creates sound waves. You can make water waves in a swimming pool by slapping the water with your hand. You can no doubt think of other types of waves. Some, such as water waves, are visible. Some, such as sound waves, are not. But *every wave is a disturbance that moves from its source and carries energy*. Other examples of waves include earthquakes and visible light. Even subatomic particles, such as electrons, can behave like waves.

By studying oscillatory motion and waves, we shall find that a small number of underlying principles describe all of them and that wave phenomena are more common than you have ever imagined. We begin by studying the type of force that underlies the simplest oscillations and waves. We will then expand our exploration of oscillatory motion and waves to

include concepts such as simple harmonic motion, uniform circular motion, and damped harmonic motion. Finally, we will explore what happens when two or more waves share the same space, in the phenomena known as superposition and interference.

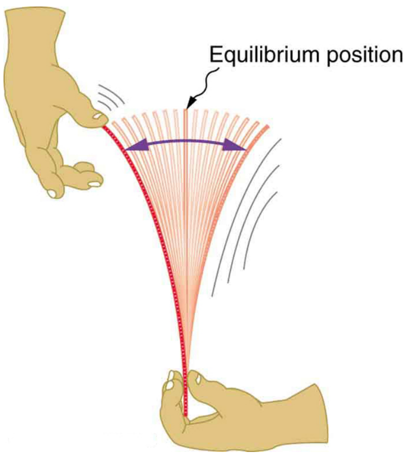
Glossary

oscillate
: moving back and forth regularly between two points

wave
: a disturbance that moves from its source and carries energy

9.2 - Hooke's Law: Revisited

- Explain Newton's third law of motion with respect to stress and deformation.
- Describe the restoration of force and displacement.
- Calculate the energy in Hook's Law of deformation, and the stored energy in a string.



When displaced from its vertical equilibrium position, this plastic ruler oscillates back and forth because of the restoring force opposing displacement. When the ruler is on the left, there is a force to the right, and vice versa.

Newton's first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line

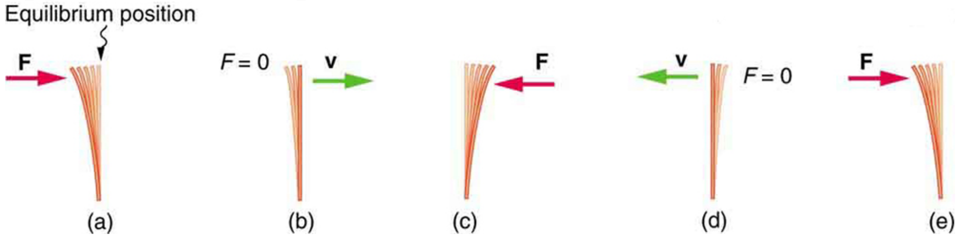
at a constant speed rather than oscillate. Consider, for example, plucking a plastic ruler to the left as shown in [link]. The deformation of the ruler creates a force in the opposite direction, known as a **restoring force**. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until dissipative forces dampen the motion. These forces remove mechanical energy from the system, gradually reducing the motion until the ruler comes to rest.

The simplest oscillations occur when the restoring force is directly proportional to displacement. When stress and strain were covered in Newton's Third Law of Motion, the name was given to this relationship between force and displacement was Hooke's law:

Equation:

$$F = -\mathrm{kx}.$$

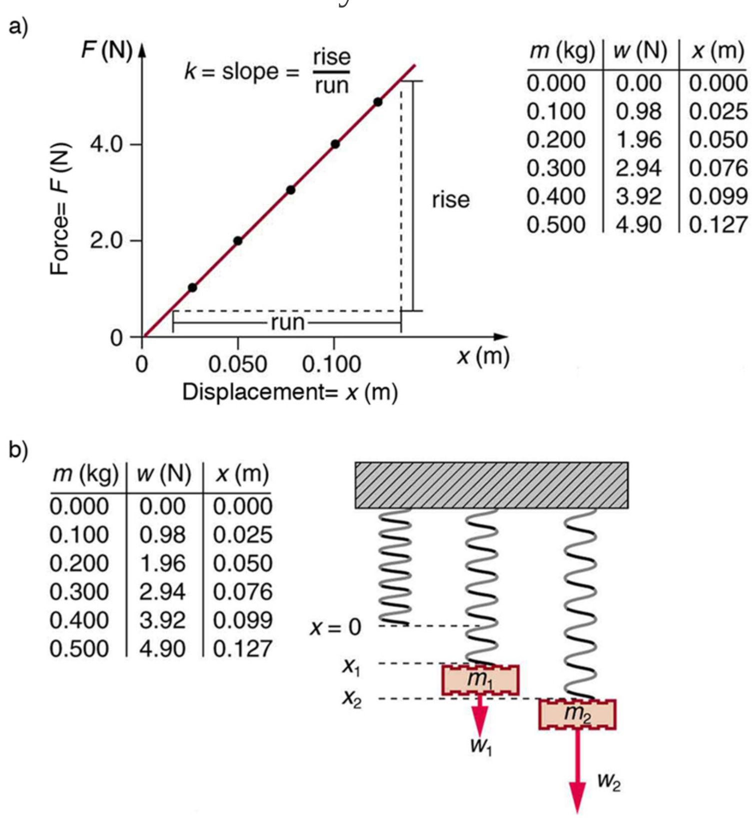
Here, F is the restoring force, x is the displacement from equilibrium or **deformation**, and k is a constant related to the difficulty in deforming the system. The minus sign indicates the restoring force is in the direction opposite to the displacement.



(a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping

(caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

The **force constant** k is related to the rigidity (or stiffness) of a system—the larger the force constant, the greater the restoring force, and the stiffer the system. The units of k are newtons per meter (N/m). For example, k is directly related to Young’s modulus when we stretch a string. [link] shows a graph of the absolute value of the restoring force versus the displacement for a system that can be described by Hooke’s law—a simple spring in this case. The slope of the graph equals the force constant k in newtons per meter. A common physics laboratory exercise is to measure restoring forces created by springs, determine if they follow Hooke’s law, and calculate their force constants if they do.



m (kg)	w (N)	x (m)
0.000	0.00	0.000
0.100	0.98	0.025
0.200	1.96	0.050
0.300	2.94	0.076
0.400	3.92	0.099
0.500	4.90	0.127

(a) A graph of absolute value of the restoring force versus displacement is

displayed. The fact that the graph is a straight line means that the system obeys Hooke’s law. The slope of the graph is the force constant k. (b) The data in the graph were generated by measuring the displacement of a spring from equilibrium while supporting various weights. The restoring force equals the weight supported, if the mass is stationary.

Example:
How Stiff Are Car Springs?

The mass of a car increases due to the introduction of a passenger. This affects the displacement of

the car on its suspension system. (credit: exfordy on Flickr)

What is the force constant for the suspension system of a car that settles 1.20 cm when an 80.0-kg person gets in?

Strategy

Consider the car to be in its equilibrium position $x = 0$ before the person gets in. The car then settles down 1.20 cm, which means it is displaced to a position $x = -1.20 \times 10^{-2}$ m. At that point, the springs supply a restoring force F equal to the person's weight $w = \text{mg} = (80.0 \text{ kg})\left(9.80 \text{ m/s}^2\right) = 784 \text{ N}$. We take this force to be F in Hooke's law. Knowing F and x, we can then solve the force constant k.

Solution

1. Solve Hooke's law, $F = -\text{kx}$, for k:
   Equation:

$$k = -\frac{F}{x}.$$

   Substitute known values and solve k:
   Equation:

$$\begin{aligned} k &= -\frac{784 \text{ N}}{-1.20 \times 10^{-2} \text{ m}} \\ &= 6.53 \times 10^{4} \text{ N/m}. \end{aligned}$$

Discussion

Note that F and x have opposite signs because they are in opposite directions—the restoring force is up, and the displacement is down. Also, note that the car would oscillate up and down when the person got in if it

were not for damping (due to frictional forces) provided by shock absorbers. Bouncing cars are a sure sign of bad shock absorbers.

Energy in Hooke’s Law of Deformation

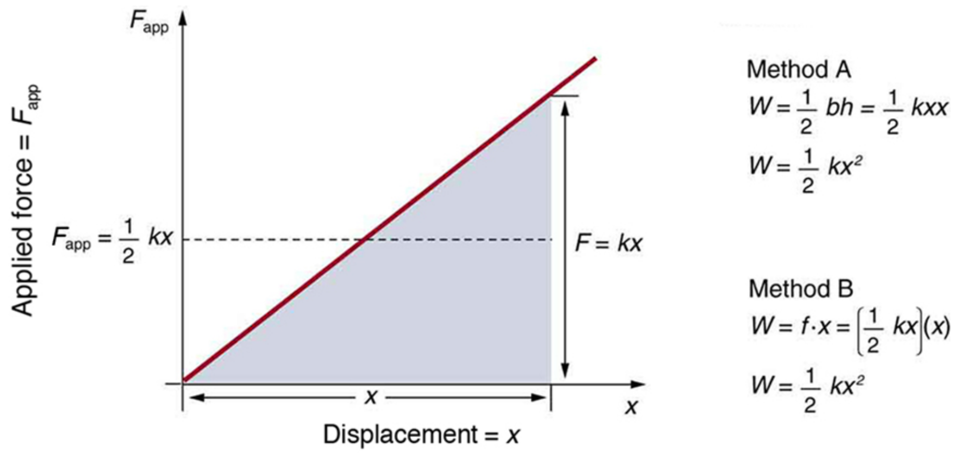
In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation, and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. The potential energy stored in a spring is $\mathrm{PE_{el}} = \frac{1}{2}\mathrm{kx}^2$. Here, we generalize the idea to elastic potential energy for a deformation of any system that can be described by Hooke’s law. Hence,

Equation:

$$\mathrm{PE_{el}} = \frac{1}{2}\mathrm{kx}^2,$$

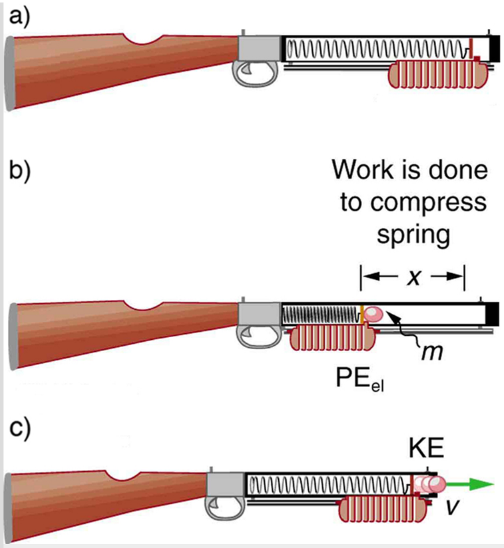
where $\mathrm{PE_{el}}$ is the **elastic potential energy** stored in any deformed system that obeys Hooke’s law and has a displacement x from equilibrium and a force constant k.

It is possible to find the work done in deforming a system in order to find the energy stored. This work is performed by an applied force F_{app}. The applied force is exactly opposite to the restoring force (action-reaction), and so $F_{\text{app}} = \mathrm{kx}$. [link] shows a graph of the applied force versus deformation x for a system that can be described by Hooke’s law. Work done on the system is force multiplied by distance, which equals the area under the curve or $(1/2)\mathrm{kx}^2$(Method A in the figure). Another way to determine the work is to note that the force increases linearly from 0 to kx, so that the average force is $(1/2)\ \mathrm{kx}$, the distance moved is x, and thus $W = F_{\text{app}}d = [(1/2)\mathrm{kx}](x) = (1/2)\mathrm{kx}^2$ (Method B in the figure).



A graph of applied force versus distance for the deformation of a system that can be described by Hooke's law is displayed. The work done on the system equals the area under the graph or the area of the triangle, which is half its base multiplied by its height, or $W = (1/2)\mathrm{kx}^2$.

Example:
Calculating Stored Energy: A Tranquilizer Gun Spring
We can use a toy gun's spring mechanism to ask and answer two simple questions: (a) How much energy is stored in the spring of a tranquilizer gun that has a force constant of 50.0 N/m and is compressed 0.150 m? (b) If you neglect friction and the mass of the spring, at what speed will a 2.00-g projectile be ejected from the gun?



(a) In this image of the gun, the spring is uncompressed before being cocked. (b) The spring has been compressed a distance x, and the projectile is in place. (c) When released, the spring converts elastic potential energy PE_{el} into kinetic energy.

Strategy for a

(a): The energy stored in the spring can be found directly from elastic potential energy equation, because k and x are given.

Solution for a

Entering the given values for k and x yields

Equation:

$$\begin{aligned} \text{PE}_{\text{el}} &= \frac{1}{2}\text{kx}^2 = \frac{1}{2}(50.0\ \text{N/m})(0.150\ \text{m})^2 = 0.563\ \text{N}\cdot\text{m} \\ &= 0.563\ \text{J} \end{aligned}$$

Strategy for b

Because there is no friction, the potential energy is converted entirely into kinetic energy. The expression for kinetic energy can be solved for the projectile's speed.

Solution for b

1. Identify known quantities:
   Equation:

$$\mathrm{KE_f} = \mathrm{PE_{el}} \ \text{ or } \ 1/2mv^2 = (1/2)kx^2 = \mathrm{PE_{el}} = 0.563\ \mathrm{J}$$

2. Solve for v:
   Equation:

$$v = \left[\frac{2\mathrm{PE_{el}}}{m}\right]^{1/2} = \left[\frac{2(0.563\ \mathrm{J})}{0.002\ \mathrm{kg}}\right]^{1/2} = 23.7(\mathrm{J/kg})^{1/2}$$

3. Convert units: $23.7\ \mathrm{m/s}$

Discussion

(a) and (b): This projectile speed is impressive for a tranquilizer gun (more than 80 km/h). The numbers in this problem seem reasonable. The force needed to compress the spring is small enough for an adult to manage, and the energy imparted to the dart is small enough to limit the damage it might do. Yet, the speed of the dart is great enough for it to travel an acceptable distance.

Exercise:
Check your Understanding

Problem:

Envision holding the end of a ruler with one hand and deforming it with the other. When you let go, you can see the oscillations of the ruler. In what way could you modify this simple experiment to increase the rigidity of the system?

Solution:
Answer

You could hold the ruler at its midpoint so that the part of the ruler that oscillates is half as long as in the original experiment.

Exercise:
Check your Understanding

Problem:

If you apply a deforming force on an object and let it come to equilibrium, what happened to the work you did on the system?

Solution:
Answer

It was stored in the object as potential energy.

Section Summary

- An oscillation is a back and forth motion of an object between two points of deformation.
- An oscillation may create a wave, which is a disturbance that propagates from where it was created.
- The simplest type of oscillations and waves are related to systems that can be described by Hooke's law:
  Equation:
  $$F = -\text{kx},$$
  where F is the restoring force, x is the displacement from equilibrium or deformation, and k is the force constant of the system.
- Elastic potential energy PE_{el} stored in the deformation of a system that can be described by Hooke's law is given by
  Equation:

$$\mathrm{PE_{el}} = (1/2)kx^2.$$

Glossary

deformation
: displacement from equilibrium

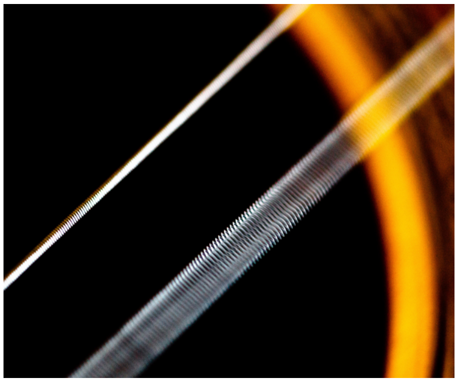
elastic potential energy
: potential energy stored as a result of deformation of an elastic object, such as the stretching of a spring

force constant
: a constant related to the rigidity of a system: the larger the force constant, the more rigid the system; the force constant is represented by *k*

restoring force
: force acting in opposition to the force caused by a deformation

9.3 - Period and Frequency in Oscillations

- Observe the vibrations of a guitar string.
- Determine the frequency of oscillations.



The strings on this guitar vibrate at regular time intervals. (credit: JAR)

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each successive vibration of the string takes the same time as the previous one. We define **periodic motion** to be a motion that repeats itself at regular time intervals, such as exhibited by the guitar string or by an object on a spring moving up and down. The time to complete one oscillation remains constant and is called the **period** T. Its units are usually seconds, but may be any convenient unit of time. The word period refers to the time for some event whether repetitive or not; but we shall be primarily interested in periodic motion, which is by definition repetitive. A concept closely related to period is the frequency of an event. For example, if you get a paycheck twice a month, the frequency of payment is two per month and the period between checks is half a month. **Frequency** f is defined to be the number of events per unit time. For periodic motion, frequency is the number of oscillations per unit time. The relationship between frequency and period is

Equation:

$$f = \frac{1}{T}.$$

The SI unit for frequency is the *cycle per second*, which is defined to be a *hertz* (Hz):

Equation:

$$1\ \text{Hz} = 1\frac{\text{cycle}}{\text{sec}}\text{ or }1\ \text{Hz} = \frac{1}{\text{s}}$$

A cycle is one complete oscillation. Note that a vibration can be a single or multiple event, whereas oscillations are usually repetitive for a significant number of cycles.

Example:
Determine the Frequency of Two Oscillations: Medical Ultrasound and the Period of Middle C

We can use the formulas presented in this module to determine both the frequency based on known oscillations and the oscillation based on a known frequency. Let’s try one example of each. (a) A medical imaging device produces ultrasound by oscillating with a period of 0.400 µs. What is the frequency of this oscillation? (b) The frequency of middle C on a typical musical instrument is 264 Hz. What is the time for one complete oscillation?

Strategy

Both questions (a) and (b) can be answered using the relationship between period and frequency. In question (a), the period T is given and we are asked to find frequency f. In question (b), the frequency f is given and we are asked to find the period T.

Solution a

1. Substitute 0.400 µs for T in $f = \frac{1}{T}$:
   Equation:

$$f = \frac{1}{T} = \frac{1}{0.400 \times 10^{-6}\text{ s}}.$$

Solve to find
Equation:

$$f = 2.50 \times 10^{6}\text{ Hz}.$$

Discussion a
The frequency of sound found in (a) is much higher than the highest frequency that humans can hear and, therefore, is called ultrasound. Appropriate oscillations at this frequency generate ultrasound used for noninvasive medical diagnoses, such as observations of a fetus in the womb.
Solution b

1. Identify the known values:

   The time for one complete oscillation is the period T:
   Equation:

   $$f = \frac{1}{T}.$$

2. Solve for T:
   Equation:

   $$T = \frac{1}{f}.$$

3. Substitute the given value for the frequency into the resulting expression:
   Equation:

   $$T = \frac{1}{f} = \frac{1}{264\text{ Hz}} = \frac{1}{264\text{ cycles/s}} = 3.79 \times 10^{-3}\text{ s} = 3.79\text{ ms}.$$

Discussion
The period found in (b) is the time per cycle, but this value is often quoted as simply the time in convenient units (ms or milliseconds in this case).

Exercise:
Check your Understanding

Problem:

Identify an event in your life (such as receiving a paycheck) that occurs regularly. Identify both the period and frequency of this event.

Solution:

I visit my parents for dinner every other Sunday. The frequency of my visits is 26 per calendar year. The period is two weeks.

Section Summary

- Periodic motion is a repetitious oscillation.
- The time for one oscillation is the period T.
- The number of oscillations per unit time is the frequency f.
- These quantities are related by
  Equation:

$$f = \frac{1}{T}.$$

Problems & Exercises

Exercise:

Problem: What is the period of 60.0 Hz electrical power?

Solution:

16.7 ms

Exercise:

Problem:

If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in units of seconds?

Solution:

0.400 s/beats

Exercise:

Problem:

A stroboscope is set to flash every 8.00×10^{-5} s. What is the frequency of the flashes?

Solution:

12,500 Hz

Glossary

period
: time it takes to complete one oscillation

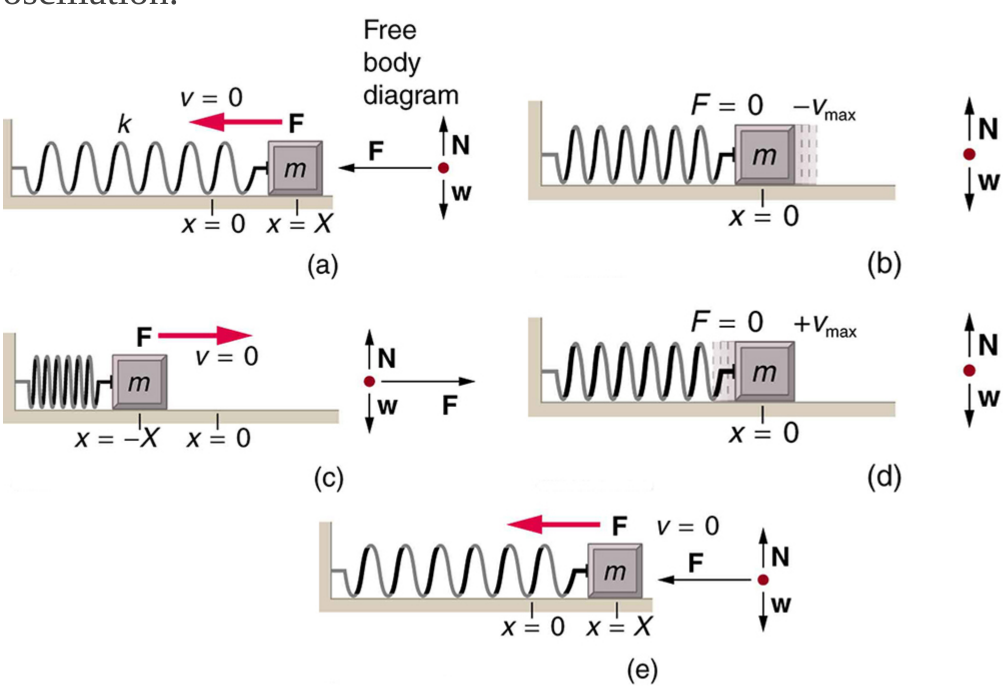
periodic motion
: motion that repeats itself at regular time intervals

frequency
: number of events per unit of time

9.4 - Simple Harmonic Motion

- Describe a simple harmonic oscillator.
- Explain the link between simple harmonic motion and waves.

The oscillations of a system in which the net force can be described by Hooke's law are of special importance, because they are very common. They are also the simplest oscillatory systems. **Simple Harmonic Motion** (SHM) is the name given to oscillatory motion for a system where the net force can be described by Hooke's law, and such a system is called a **simple harmonic oscillator**. If the net force can be described by Hooke's law and there is no *damping* (by friction or other non-conservative forces), then a simple harmonic oscillator will oscillate with equal displacement on either side of the equilibrium position, as shown for an object on a spring in [link]. The maximum displacement from equilibrium is called the **amplitude** X. The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, the units of amplitude and displacement are meters; whereas for sound oscillations, they have units of pressure (and other types of oscillations have yet other units). Because amplitude is the maximum displacement, it is related to the energy in the oscillation.



An object attached to a spring sliding on a frictionless surface is an uncomplicated simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude X and a period T. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T.

What is so significant about simple harmonic motion? One special thing is that the period T and frequency f of a simple harmonic oscillator are independent of amplitude. The string of a guitar, for example, will oscillate with the same frequency whether plucked gently or hard. Because the period is constant, a simple harmonic oscillator can be used as a clock.

Two important factors do affect the period of a simple harmonic oscillator. The period is related to how stiff the system is. A very stiff object has a large force constant k, which causes the system to have a smaller period. For example, you can adjust a diving board's stiffness—the stiffer it is, the faster it vibrates, and the shorter its period. Period also depends on the mass of the oscillating system. The more massive the system is, the longer the period. For example, a heavy person on a diving board bounces up and down more slowly than a light one.

In fact, the mass m and the force constant k are the *only* factors that affect the period and frequency of simple harmonic motion.

Note:
Period of Simple Harmonic Oscillator
The *period of a simple harmonic oscillator* is given by
Equation:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

and, because $f = 1/T$, the *frequency of a simple harmonic oscillator* is
Equation:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

Note that neither T nor f has any dependence on amplitude.

Example:
Calculate the Frequency and Period of Oscillations: Bad Shock Absorbers in a Car
If the shock absorbers in a car go bad, then the car will oscillate at the least provocation, such as when going over bumps in the road and after stopping (See [link]). Calculate the frequency and period of these oscillations for such a car if the car's mass (including its load) is 900 kg and the force constant (k) of the suspension system is $6.53 \times 10^4\ \text{N/m}$.
Strategy
The frequency of the car's oscillations will be that of a simple harmonic oscillator as given in the equation $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. The mass and the force constant are both given.
Solution

1. Enter the known values of *k* and *m*:
   Equation:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{6.53 \times 10^4\ \text{N/m}}{900\ \text{kg}}}.$$

2. Calculate the frequency:
   Equation:

$$\frac{1}{2\pi}\sqrt{72.6/\text{s}^{-2}} = 1.3656/\text{s}^{-1} \approx 1.36/\text{s}^{-1} = 1.36\text{ Hz}.$$

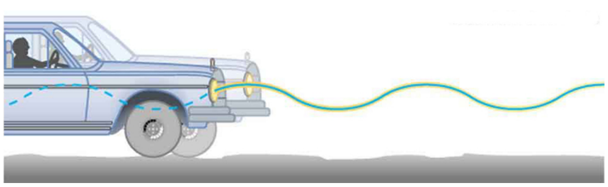
3. You could use $T = 2\pi\sqrt{\frac{m}{k}}$ to calculate the period, but it is simpler to use the relationship $T = 1/f$ and substitute the value just found for f:
   Equation:

$$T = \frac{1}{f} = \frac{1}{1.356\text{ Hz}} = 0.738\text{ s}.$$

Discussion
The values of T and f both seem about right for a bouncing car. You can observe these oscillations if you push down hard on the end of a car and let go.

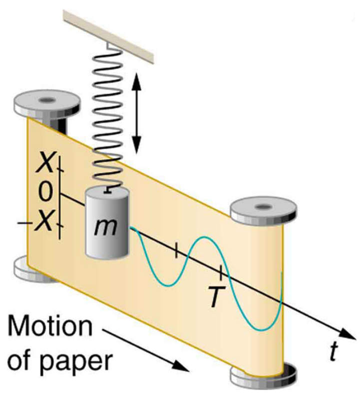
The Link between Simple Harmonic Motion and Waves

If a time-exposure photograph of the bouncing car were taken as it drove by, the headlight would make a wavelike streak, as shown in [link]. Similarly, [link] shows an object bouncing on a spring as it leaves a wavelike "trace of its position on a moving strip of paper. Both waves are sine functions. All simple harmonic motion is intimately related to sine and cosine waves.



The bouncing car makes a wavelike motion. If the restoring force in the suspension system can be described only by

Hooke’s law, then the wave is a sine function. (The wave is the trace produced by the headlight as the car moves to the right.)



The vertical position of an object bouncing on a spring is recorded on a strip of moving paper, leaving a sine wave.

The displacement as a function of time *t* in any simple harmonic motion—that is, one in which the net restoring force can be described by Hooke’s law, is given by

Equation:

$$x(t) = X\cos\frac{2\pi t}{T},$$

where X is amplitude. At $t=0$, the initial position is $x_0=X$, and the displacement oscillates back and forth with a period T. (When $t=T$, we get $x=X$ again because cos $2\pi=1$.). Furthermore, from this expression for x, the velocity v as a function of time is given by:

Equation:

$$v(t)=-v_{\text{max}}\sin\left(\frac{2\pi t}{T}\right),$$

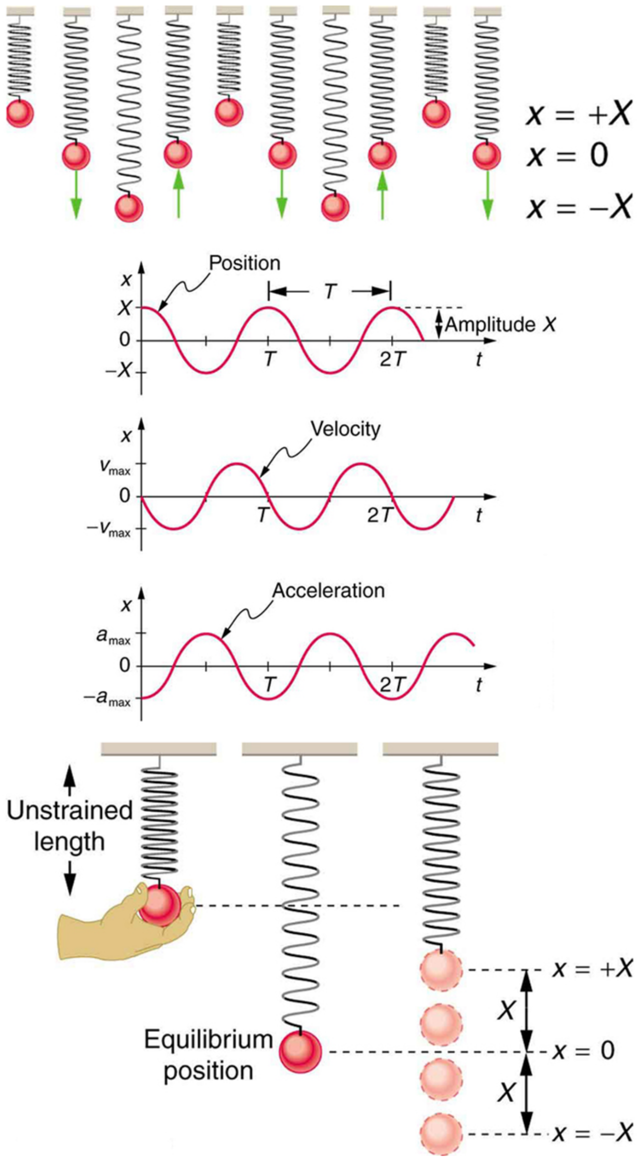
where $v_{\text{max}}=2\pi X/T=X\sqrt{k/m}$. The object has zero velocity at maximum displacement—for example, $v=0$ when $t=0$, and at that time $x=X$. The minus sign in the first equation for $v(t)$ gives the correct direction for the velocity. Just after the start of the motion, for instance, the velocity is negative because the system is moving back toward the equilibrium point. Finally, we can get an expression for acceleration using Newton's second law. [Then we have $x(t)$, $v(t)$, t, and $a(t)$, the quantities needed for kinematics and a description of simple harmonic motion.] According to Newton's second law, the acceleration is $a=F/m=\text{kx}/m$. So, $a(t)$ is also a cosine function:

Equation:

$$a(t)=-\frac{\text{kX}}{m}\cos\frac{2\pi t}{T}.$$

Hence, $a(t)$ is directly proportional to and in the opposite direction to $x(t)$.

[link] shows the simple harmonic motion of an object on a spring and presents graphs of $x(t)$,$v(t\ )$, and $a(t)$ versus time.



Graphs of $x(t)$, $v(t)$, and $a(t)$ versus t for the motion of an object on a spring. The net force on the object can be described by Hooke's law, and so the object undergoes simple harmonic motion. Note that the initial position has the vertical displacement at its maximum value X; v is initially zero and then negative as the object moves down; and the initial acceleration

is negative, back toward the equilibrium position and becomes zero at that point.

The most important point here is that these equations are mathematically straightforward and are valid for all simple harmonic motion. They are very useful in visualizing waves associated with simple harmonic motion, including visualizing how waves add with one another.

Exercise:
Check Your Understanding

Problem:

Suppose you pluck a banjo string. You hear a single note that starts out loud and slowly quiets over time. Describe what happens to the sound waves in terms of period, frequency and amplitude as the sound decreases in volume.

Solution:

Frequency and period remain essentially unchanged. Only amplitude decreases as volume decreases.

Exercise:
Check Your Understanding

Problem:

A babysitter is pushing a child on a swing. At the point where the swing reaches x, where would the corresponding point on a wave of this motion be located?

Solution:

x is the maximum deformation, which corresponds to the amplitude of the wave. The point on the wave would either be at the very top or the very bottom of the curve.

Section Summary

- Simple harmonic motion is oscillatory motion for a system that can be described only by Hooke's law. Such a system is also called a simple harmonic oscillator.
- Maximum displacement is the amplitude X. The period T and frequency f of a simple harmonic oscillator are given by

  $T = 2\pi\sqrt{\frac{m}{k}}$ and $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where m is the mass of the system.
- Displacement in simple harmonic motion as a function of time is given by $x(t) = X\cos\frac{2\pi t}{T}$.
- The velocity is given by $v(t) = -v_{\text{max}}\sin\frac{2\pi t}{T}$, where $v_{\text{max}} = \sqrt{k/m}X$.
- The acceleration is found to be $a(t) = -\frac{kX}{m}\cos\frac{2\pi t}{T}$.

Conceptual Questions

Exercise:

Problem:

What conditions must be met to produce simple harmonic motion?

Exercise:

Problem:

(a) If frequency is not constant for some oscillation, can the oscillation be simple harmonic motion?

(b) Can you think of any examples of harmonic motion where the frequency may depend on the amplitude?

Exercise:

Problem:

Give an example of a simple harmonic oscillator, specifically noting how its frequency is independent of amplitude.

Problems & Exercises

Exercise:

Problem:

If the spring constant of a simple harmonic oscillator is doubled, by what factor will the mass of the system need to change in order for the frequency of the motion to remain the same?

Exercise:

Problem:

A 0.500-kg mass suspended from a spring oscillates with a period of 1.50 s. How much mass must be added to the object to change the period to 2.00 s?

Solution:

0.389 kg

Exercise:

Problem:

A diver on a diving board is undergoing simple harmonic motion. Her mass is 55.0 kg and the period of her motion is 0.800 s. The next diver is a male whose period of simple harmonic oscillation is 1.05 s. What is his mass if the mass of the board is negligible?

Solution:

94.7 kg

Exercise:

Problem:

A 90.0-kg skydiver hanging from a parachute bounces up and down with a period of 1.50 s. What is the new period of oscillation when a second skydiver, whose mass is 60.0 kg, hangs from the legs of the first, as seen in [link].

The oscillations of one skydiver are about to be affected by a second skydiver. (credit: U.S. Army, www.army.mil)

Solution:

1.94 s

Glossary

amplitude
: the maximum displacement from the equilibrium position of an object oscillating around the equilibrium position

simple harmonic motion

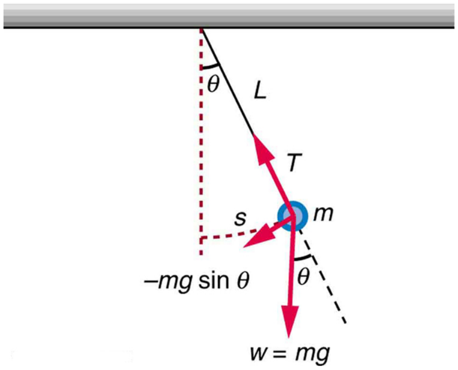
the oscillatory motion in a system where the net force can be described by Hooke’s law

simple harmonic oscillator

a device that implements Hooke’s law, such as a mass that is attached to a spring, with the other end of the spring being connected to a rigid support such as a wall

9.5 - The Simple Pendulum

- Measure acceleration due to gravity.



A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch appreciably. The linear displacement from equilibrium is s, the length of the arc. Also shown are the forces on the bob, which result in a net force of $-\mathrm{mg}\ \sin\theta$ toward the equilibrium position—that is, a restoring force.

Pendulums are in common usage. Some have crucial uses, such as in clocks; some are for fun, such as a child's swing; and some are just there, such as the sinker on a fishing line. For small displacements, a pendulum is a simple harmonic oscillator. A **simple pendulum** is defined to have an

object that has a small mass, also known as the pendulum bob, which is suspended from a light wire or string, such as shown in [link]. Exploring the simple pendulum a bit further, we can discover the conditions under which it performs simple harmonic motion, and we can derive an interesting expression for its period.

We begin by defining the displacement to be the arc length s. We see from [link] that the net force on the bob is tangent to the arc and equals $-\text{mg}\sin\theta$. (The weight mg has components $\text{mg}\cos\theta$ along the string and $\text{mg}\sin\theta$ tangent to the arc.) Tension in the string exactly cancels the component $\text{mg}\cos\theta$ parallel to the string. This leaves a *net* restoring force back toward the equilibrium position at $\theta = 0$.

Now, if we can show that the restoring force is directly proportional to the displacement, then we have a simple harmonic oscillator. In trying to determine if we have a simple harmonic oscillator, we should note that for small angles (less than about 15º), $\sin\theta \approx \theta$ ($\sin\theta$ and θ differ by about 1% or less at smaller angles). Thus, for angles less than about 15º, the restoring force F is

Equation:

$$F \approx -\text{mg}\theta.$$

The displacement s is directly proportional to θ. When θ is expressed in radians, the arc length in a circle is related to its radius (L in this instance) by:

Equation:

$$s = L\theta,$$

so that

Equation:

$$\theta = \frac{s}{L}.$$

For small angles, then, the expression for the restoring force is:
Equation:

$$F \approx -\frac{\mathrm{mg}}{L}s$$

This expression is of the form:
Equation:

$$F = -\mathrm{kx},$$

where the force constant is given by $k = mg/L$ and the displacement is given by $x = s$. For angles less than about 15^{o}, the restoring force is directly proportional to the displacement, and the simple pendulum is a simple harmonic oscillator.

Using this equation, we can find the period of a pendulum for amplitudes less than about 15^{o}. For the simple pendulum:
Equation:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\mathrm{mg}/L}}.$$

Thus,
Equation:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

for the period of a simple pendulum. This result is interesting because of its simplicity. The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass. As with simple harmonic oscillators, the period T for a pendulum is nearly independent of amplitude,

especially if θ is less than about 15º. Even simple pendulum clocks can be finely adjusted and accurate.

Note the dependence of T on g. If the length of a pendulum is precisely known, it can actually be used to measure the acceleration due to gravity. Consider the following example.

Example:
Measuring Acceleration due to Gravity: The Period of a Pendulum
What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?
Strategy
We are asked to find g given the period T and the length L of a pendulum. We can solve $T = 2\pi\sqrt{\frac{L}{g}}$ for g, assuming only that the angle of deflection is less than $15°$.
Solution

1. Square $T = 2\pi\sqrt{\frac{L}{g}}$ and solve for g:
   Equation:
   $$g = 4\pi^2 \frac{L}{T^2}.$$
2. Substitute known values into the new equation:
   Equation:
   $$g = 4\pi^2 \frac{0.75000 \text{ m}}{(1.7357 \text{ s})^2}.$$
3. Calculate to find g:
   Equation:
   $$g = 9.8281 \text{ m/s}^2.$$

Discussion

This method for determining g can be very accurate. This is why length and period are given to five digits in this example. For the precision of the approximation $\sin\theta \approx \theta$ to be better than the precision of the pendulum length and period, the maximum displacement angle should be kept below about 0.5°.

Note:

Making Career Connections

Knowing g can be important in geological exploration; for example, a map of g over large geographical regions aids the study of plate tectonics and helps in the search for oil fields and large mineral deposits.

Exercise:

Check Your Understanding

Problem:

An engineer builds two simple pendula. Both are suspended from small wires secured to the ceiling of a room. Each pendulum hovers 2 cm above the floor. Pendulum 1 has a bob with a mass of 10 kg. Pendulum 2 has a bob with a mass of 100 kg. Describe how the motion of the pendula will differ if the bobs are both displaced by 12°.

Solution:

The movement of the pendula will not differ at all because the mass of the bob has no effect on the motion of a simple pendulum. The pendula are only affected by the period (which is related to the pendulum's length) and by the acceleration due to gravity.

Section Summary

- A mass m suspended by a wire of length L is a simple pendulum and undergoes simple harmonic motion for amplitudes less than about 15^{o}.

  The period of a simple pendulum is
  Equation:

  $$T = 2\pi\sqrt{\frac{L}{g}},$$

  where L is the length of the string and g is the acceleration due to gravity.

Conceptual Questions

Exercise:

Problem:

Pendulum clocks are made to run at the correct rate by adjusting the pendulum's length. Suppose you move from one city to another where the acceleration due to gravity is slightly greater, taking your pendulum clock with you, will you have to lengthen or shorten the pendulum to keep the correct time, other factors remaining constant? Explain your answer.

Problems & Exercises

As usual, the acceleration due to gravity in these problems is taken to be $g = 9.80 \text{ m/s}^2$, unless otherwise specified.
Exercise:

Problem:

What is the length of a pendulum that has a period of 0.500 s?

Solution:

6.21 cm

Exercise:

Problem: What is the period of a 1.00-m-long pendulum?

Solution:

2.01 s

Exercise:

Problem:

A pendulum with a period of 2.00000 s in one location $\left(g = 9.80\ \text{m/s}^2\right)$ is moved to a new location where the period is now 1.99796 s. What is the acceleration due to gravity at its new location?

Exercise:

Problem:

(a) What is the effect on the period of a pendulum if you double its length?

(b) What is the effect on the period of a pendulum if you decrease its length by 5.00%?

Solution:

(a) Period increases by a factor of 1.41 ($\sqrt{2}$)

(b) Period decreases to 97.5% of old period

Exercise:

Problem:

At what rate will a pendulum clock run on the Moon, where the acceleration due to gravity is $1.63\ \mathrm{m/s^2}$, if it keeps time accurately on Earth? That is, find the time (in hours) it takes the clock's hour hand to make one revolution on the Moon.

Solution:

Slow by a factor of 2.45

Glossary

simple pendulum
: an object with a small mass suspended from a light wire or string

9.6 - Energy and the Simple Harmonic Oscillator

- Determine the maximum speed of an oscillating system.

To study the energy of a simple harmonic oscillator, we first consider all the forms of energy it can have We know from Hooke's Law: Stress and Strain Revisited that the energy stored in the deformation of a simple harmonic oscillator is a form of potential energy given by:
Equation:

$$\mathrm{PE_{el}} = \frac{1}{2}kx^2.$$

Because a simple harmonic oscillator has no dissipative forces, the other important form of energy is kinetic energy KE. Conservation of energy for these two forms is:
Equation:

$$\mathrm{KE} + \mathrm{PE_{el}} = \mathrm{constant}$$

or
Equation:

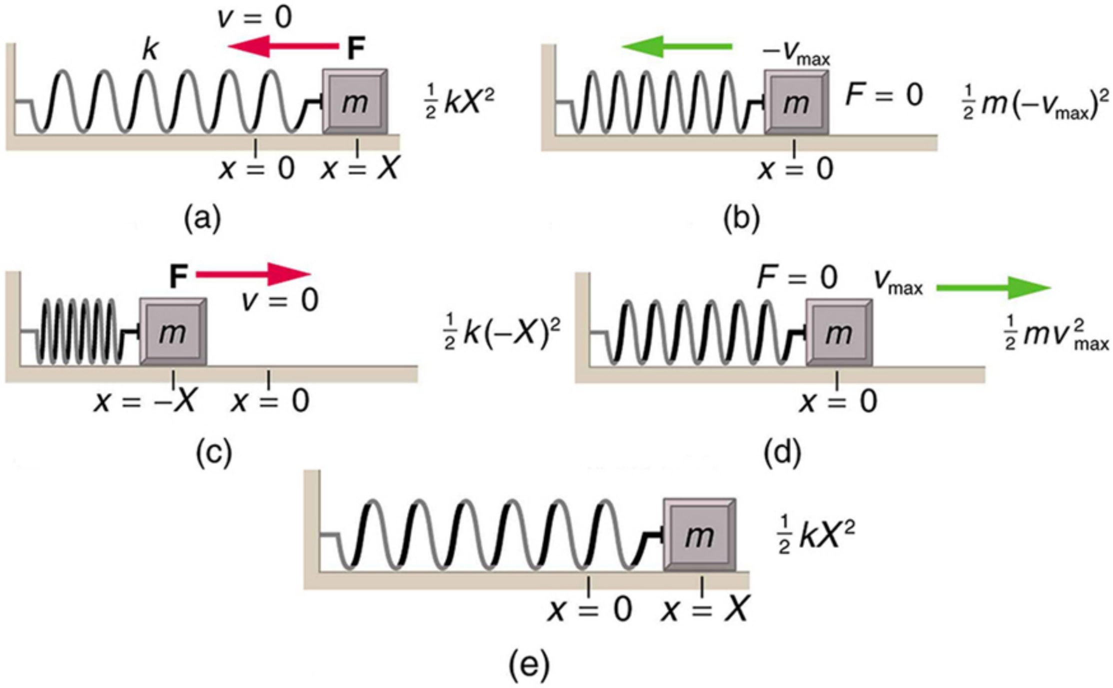
$$\frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}\mathrm{kx}^2 = \mathrm{constant}.$$

This statement of conservation of energy is valid for *all* simple harmonic oscillators, including ones where the gravitational force plays a role

Namely, for a simple pendulum we replace the velocity with $v = L\omega$, the spring constant with $k = \mathrm{mg}/L$, and the displacement term with $x = L\theta$. Thus
Equation:

$$\frac{1}{2}\mathrm{mL}^2\omega^2 + \frac{1}{2}\mathrm{mgL}\theta^2 = \mathrm{constant}.$$

In the case of undamped simple harmonic motion, the energy oscillates back and forth between kinetic and potential, going completely from one to the other as the system oscillates. So for the simple example of an object on a frictionless surface attached to a spring, as shown again in [link], the motion starts with all of the energy stored in the spring. As the object starts to move, the elastic potential energy is converted to kinetic energy, becoming entirely kinetic energy at the equilibrium position. It is then converted back into elastic potential energy by the spring, the velocity becomes zero when the kinetic energy is completely converted, and so on. This concept provides extra insight here and in later applications of simple harmonic motion, such as alternating current circuits.



The transformation of energy in simple harmonic motion is illustrated for an object attached to a spring on a frictionless surface.

The conservation of energy principle can be used to derive an expression for velocity v. If we start our simple harmonic motion with zero velocity and maximum displacement ($x = X$), then the total energy is

Equation:

$$\frac{1}{2}\mathrm{kX}^2.$$

This total energy is constant and is shifted back and forth between kinetic energy and potential energy, at most times being shared by each. The conservation of energy for this system in equation form is thus:

Equation:

$$\frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}\mathrm{kx}^2 = \frac{1}{2}\mathrm{kX}^2.$$

Solving this equation for v yields:

Equation:

$$v = \pm\sqrt{\frac{k}{m}(X^2 - x^2)}.$$

Manipulating this expression algebraically gives:

Equation:

$$v = \pm\sqrt{\frac{k}{m}}X\sqrt{1 - \frac{x^2}{X^2}}$$

and so

Equation:

$$v = \pm v_{\text{max}}\sqrt{1 - \frac{x^2}{X^2}},$$

where

Equation:

$$v_{\text{max}} = \sqrt{\frac{k}{m}}X.$$

From this expression, we see that the velocity is a maximum (v_{max}) at $x = 0$, as stated earlier in $v(t) = -v_{\text{max}} \sin \frac{2\pi t}{T}$. Notice that the maximum velocity depends on three factors. Maximum velocity is directly proportional to amplitude. As you might guess, the greater the maximum displacement the greater the maximum velocity. Maximum velocity is also greater for stiffer systems, because they exert greater force for the same displacement. This observation is seen in the expression for v_{max}; it is proportional to the square root of the force constant k. Finally, the maximum velocity is smaller for objects that have larger masses, because the maximum velocity is inversely proportional to the square root of m. For a given force, objects that have large masses accelerate more slowly.

A similar calculation for the simple pendulum produces a similar result, namely:

Equation:

$$\omega_{\text{max}} = \sqrt{\frac{g}{L}}\theta_{\text{max}}.$$

Example:

Determine the Maximum Speed of an Oscillating System: A Bumpy Road

Suppose that a car is 900 kg and has a suspension system that has a force constant $k = 6.53 \times 10^4\ \text{N/m}$. The car hits a bump and bounces with an amplitude of 0.100 m. What is its maximum vertical velocity if you assume no damping occurs?

Strategy

We can use the expression for v_{max} given in $v_{\text{max}} = \sqrt{\frac{k}{m}}X$ to determine the maximum vertical velocity. The variables m and k are given in the

problem statement, and the maximum displacement X is 0.100 m.
Solution

1. Identify known.
2. Substitute known values into $v_{\text{max}} = \sqrt{\frac{k}{m}}X$:

   Equation:

$$v_{\text{max}} = \sqrt{\frac{6.53 \times 10^4 \text{ N/m}}{900 \text{ kg}}}(0.100 \text{ m}).$$

3. Calculate to find v_{max}= 0.852 m/s.

Discussion
This answer seems reasonable for a bouncing car. There are other ways to use conservation of energy to find v_{max}. We could use it directly, as was done in the example featured in Hooke’s Law: Stress and Strain Revisited.
The small vertical displacement y of an oscillating simple pendulum, starting from its equilibrium position, is given as
Equation:

$$y(t) = a \sin \omega t,$$

where a is the amplitude, ω is the angular velocity and t is the time taken. Substituting $\omega = \frac{2\pi}{T}$, we have
Equation:

$$yt = a \sin\left(\frac{2\pi t}{T}\right).$$

Thus, the displacement of pendulum is a function of time as shown above. Also the velocity of the pendulum is given by
Equation:

$$v(t) = \frac{2a\pi}{T} \cos\left(\frac{2\pi t}{T}\right),$$

so the motion of the pendulum is a function of time.

Exercise:
Check Your Understanding

Problem:

Why does it hurt more if your hand is snapped with a ruler than with a loose spring, even if the displacement of each system is equal?

Solution:

The ruler is a stiffer system, which carries greater force for the same amount of displacement. The ruler snaps your hand with greater force, which hurts more.

Exercise:
Check Your Understanding

Problem:

You are observing a simple harmonic oscillator. Identify one way you could decrease the maximum velocity of the system.

Solution:

You could increase the mass of the object that is oscillating.

Section Summary

- Energy in the simple harmonic oscillator is shared between elastic potential energy and kinetic energy, with the total being constant:
  Equation:

$$\frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}\mathrm{kx}^2 = \text{constant}.$$

- Maximum velocity depends on three factors: it is directly proportional to amplitude, it is greater for stiffer systems, and it is smaller for objects that have larger masses:
  Equation:

$$v_{\text{max}} = \sqrt{\frac{k}{m}}X.$$

Problems & Exercises

Exercise:

Problem:

The length of nylon rope from which a mountain climber is suspended has a force constant of $1.40 \times 10^4 \text{ N/m}$.

(a) What is the frequency at which he bounces, given his mass plus and the mass of his equipment are 90.0 kg?

(b) How much would this rope stretch to break the climber's fall if he free-falls 2.00 m before the rope runs out of slack? Hint: Use conservation of energy.

(c) Repeat both parts of this problem in the situation where twice this length of nylon rope is used.

Solution:

(a) 1.99 Hz

(b) 50.2 cm

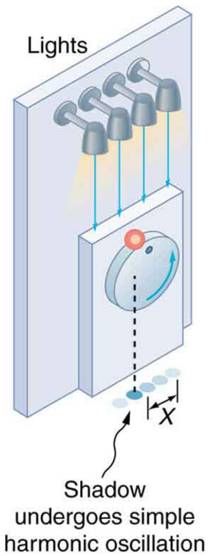
(c) 1.41 Hz, 0.710 m

9.7 - Uniform Circular Motion and Simple Harmonic Motion

- Compare simple harmonic motion with uniform circular motion.

The horses on this merry-go-round exhibit uniform circular motion. (credit: Wonderlane, Flickr)

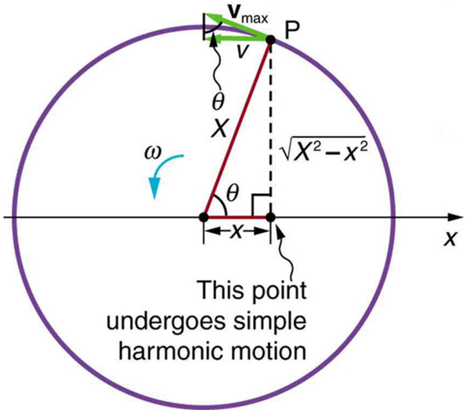
There is an easy way to produce simple harmonic motion by using uniform circular motion. [link] shows one way of using this method. A ball is attached to a uniformly rotating vertical turntable, and its shadow is projected on the floor as shown. The shadow undergoes simple harmonic motion. Hooke's law usually describes uniform circular motions (ω constant) rather than systems that have large visible displacements. So observing the projection of uniform circular motion, as in [link], is often easier than observing a precise large-scale simple harmonic oscillator. If studied in sufficient depth, simple harmonic motion produced in this manner can give considerable insight into many aspects of oscillations and waves and is very useful mathematically. In our brief treatment, we shall indicate some of the major features of this relationship and how they might be useful.



The shadow of a ball rotating at constant angular velocity ω on a turntable goes back and forth in precise simple harmonic motion.

[link] shows the basic relationship between uniform circular motion and simple harmonic motion. The point P travels around the circle at constant angular velocity ω. The point P is analogous to an object on the merry-go-

round. The projection of the position of P onto a fixed axis undergoes simple harmonic motion and is analogous to the shadow of the object. At the time shown in the figure, the projection has position x and moves to the left with velocity v. The velocity of the point P around the circle equals $v_{\max}$.The projection of $v_{\max}$ on the x-axis is the velocity v of the simple harmonic motion along the x-axis.



A point P moving on a circular path with a constant angular velocity ω is undergoing uniform circular motion. Its projection on the x-axis undergoes simple harmonic motion. Also shown is the velocity of this point around the circle, $v_{\max}$, and its projection, which is v. Note that these velocities form a similar triangle to the displacement triangle.

To see that the projection undergoes simple harmonic motion, note that its position x is given by

Equation:

$$x = X\cos\theta,$$

where $\theta = \omega t$, ω is the constant angular velocity, and X is the radius of the circular path. Thus,

Equation:

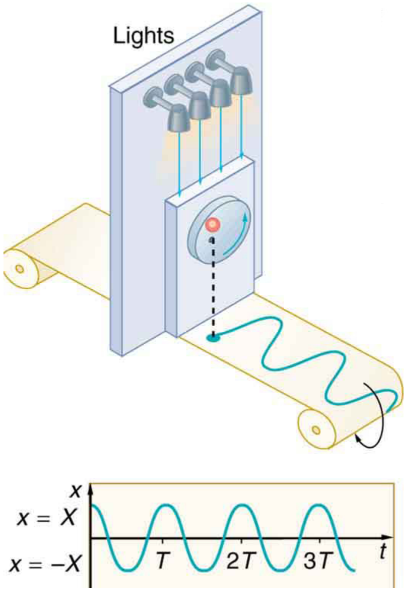
$$x = X\cos\omega t.$$

The angular velocity ω is in radians per unit time; in this case 2π radians is the time for one revolution T. That is, $\omega = 2\pi/T$. Substituting this expression for ω, we see that the position x is given by:

Equation:

$$x(t) = \cos\left(\frac{2\pi t}{T}\right).$$

This expression is the same one we had for the position of a simple harmonic oscillator in Simple Harmonic Motion: A Special Periodic Motion. If we make a graph of position versus time as in [link], we see again the wavelike character (typical of simple harmonic motion) of the projection of uniform circular motion onto the x-axis.



The position of the projection of uniform circular motion performs simple harmonic motion, as this wavelike graph of x versus t indicates.

Now let us use [link] to do some further analysis of uniform circular motion as it relates to simple harmonic motion. The triangle formed by the velocities in the figure and the triangle formed by the displacements (X, x, and $\sqrt{X^2 - x^2}$) are similar right triangles. Taking ratios of similar sides, we see that

Equation:

$$\frac{v}{v_{\text{max}}} = \frac{\sqrt{X^2 - x^2}}{X} = \sqrt{1 - \frac{x^2}{X^2}}.$$

We can solve this equation for the speed v or
Equation:

$$v = v_{\text{max}}\sqrt{1 - \frac{x^2}{X^2}}.$$

This expression for the speed of a simple harmonic oscillator is exactly the same as the equation obtained from conservation of energy considerations in Energy and the Simple Harmonic Oscillator.You can begin to see that it is possible to get all of the characteristics of simple harmonic motion from an analysis of the projection of uniform circular motion.

Finally, let us consider the period T of the motion of the projection. This period is the time it takes the point P to complete one revolution. That time is the circumference of the circle $2\pi X$ divided by the velocity around the circle, v_{max}. Thus, the period T is
Equation:

$$T = \frac{2\pi X}{v_{\text{max}}}.$$

We know from conservation of energy considerations that
Equation:

$$v_{\text{max}} = \sqrt{\frac{k}{m}}X.$$

Solving this equation for X/v_{max} gives
Equation:

$$\frac{X}{v_{\text{max}}} = \sqrt{\frac{m}{k}}.$$

Substituting this expression into the equation for T yields

Equation:

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

Thus, the period of the motion is the same as for a simple harmonic oscillator. We have determined the period for any simple harmonic oscillator using the relationship between uniform circular motion and simple harmonic motion.

Some modules occasionally refer to the connection between uniform circular motion and simple harmonic motion. Moreover, if you carry your study of physics and its applications to greater depths, you will find this relationship useful. It can, for example, help to analyze how waves add when they are superimposed.

Exercise:

Check Your Understanding

Problem:

Identify an object that undergoes uniform circular motion. Describe how you could trace the simple harmonic motion of this object as a wave.

Solution:

A record player undergoes uniform circular motion. You could attach dowel rod to one point on the outside edge of the turntable and attach a pen to the other end of the dowel. As the record player turns, the pen will move. You can drag a long piece of paper under the pen, capturing its motion as a wave.

Section Summary

A projection of uniform circular motion undergoes simple harmonic oscillation.

Problems & Exercises

Exercise:

Problem:

(a)What is the maximum velocity of an 85.0-kg person bouncing on a bathroom scale having a force constant of $1.50 \times 10^6\ \mathrm{N/m}$, if the amplitude of the bounce is 0.200 cm? (b)What is the maximum energy stored in the spring?

Solution:

a). 0.266 m/s

b). 3.00 J

Exercise:

Problem:

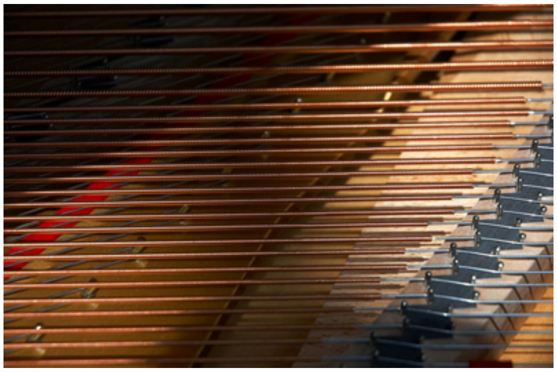
At what positions is the speed of a simple harmonic oscillator half its maximum? That is, what values of x/X give $v = \pm v_{\mathrm{max}}/2$, where X is the amplitude of the motion?

Solution:

$\pm\frac{\sqrt{3}}{2}$

9.8 - Forced Oscillations and Resonance

- Observe resonance of a paddle ball on a string.
- Observe amplitude of a damped harmonic oscillator.

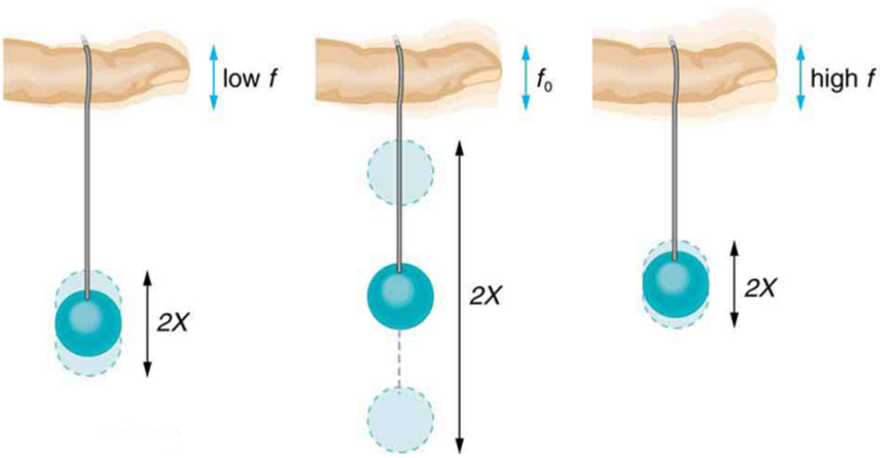


You can cause the strings in a piano to vibrate simply by producing sound waves from your voice. (credit: Matt Billings, Flickr)

Sit in front of a piano sometime and sing a loud brief note at it with the dampers off its strings. It will sing the same note back at you—the strings, having the same frequencies as your voice, are resonating in response to the forces from the sound waves that you sent to them. Your voice and a piano's strings is a good example of the fact that objects—in this case, piano strings—can be forced to oscillate but oscillate best at their natural frequency. In this section, we shall briefly explore applying a *periodic driving force* acting on a simple harmonic oscillator. The driving force puts energy into the system at a certain frequency, not necessarily the same as the natural frequency of the system. The **natural frequency** is the frequency at which a system would oscillate if there were no driving and no damping force.

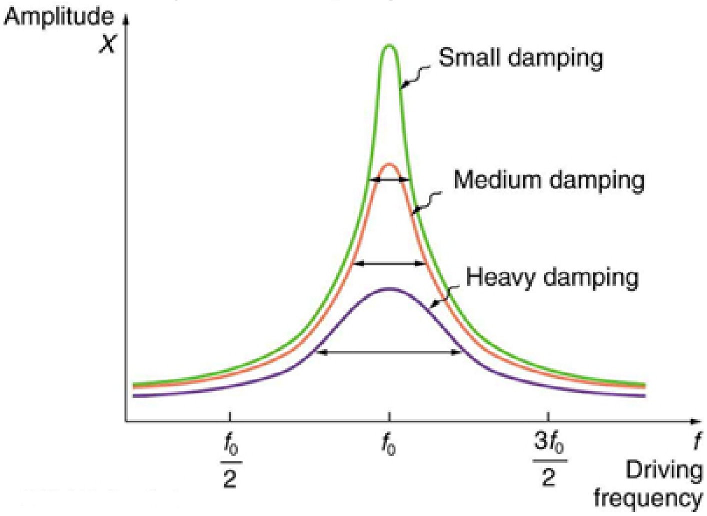
Most of us have played with toys involving an object supported on an elastic band, something like the paddle ball suspended from a finger in [link]. Imagine the finger in the figure is your finger. At first you hold your

finger steady, and the ball bounces up and down with a small amount of damping. If you move your finger up and down slowly, the ball will follow along without bouncing much on its own. As you increase the frequency at which you move your finger up and down, the ball will respond by oscillating with increasing amplitude. When you drive the ball at its natural frequency, the ball's oscillations increase in amplitude with each oscillation for as long as you drive it. The phenomenon of driving a system with a frequency equal to its natural frequency is called **resonance**. A system being driven at its natural frequency is said to **resonate**. As the driving frequency gets progressively higher than the resonant or natural frequency, the amplitude of the oscillations becomes smaller, until the oscillations nearly disappear and your finger simply moves up and down with little effect on the ball.



The paddle ball on its rubber band moves in response to the finger supporting it. If the finger moves with the natural frequency f_0 of the ball on the rubber band, then a resonance is achieved, and the amplitude of the ball's oscillations increases dramatically. At higher and lower driving frequencies, energy is transferred to the ball less efficiently, and it responds with lower-amplitude oscillations.

[link] shows a graph of the amplitude of a damped harmonic oscillator as a function of the frequency of the periodic force driving it. There are three curves on the graph, each representing a different amount of damping. All three curves peak at the point where the frequency of the driving force equals the natural frequency of the harmonic oscillator. The highest peak, or greatest response, is for the least amount of damping, because less energy is removed by the damping force.



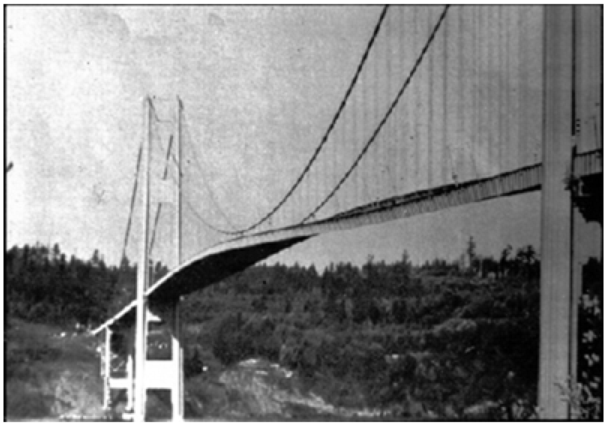
Amplitude of a harmonic oscillator as a function of the frequency of the driving force. The curves represent the same oscillator with the same natural frequency but with different amounts of damping. Resonance occurs when the driving frequency equals the natural frequency, and the greatest response is for the least amount of damping. The narrowest response is also for the least damping.

It is interesting that the widths of the resonance curves shown in [link] depend on damping: the less the damping, the narrower the resonance. The message is that if you want a driven oscillator to resonate at a very specific frequency, you need as little damping as possible. Little damping is the case

for piano strings and many other musical instruments. Conversely, if you want small-amplitude oscillations, such as in a car's suspension system, then you want heavy damping. Heavy damping reduces the amplitude, but the tradeoff is that the system responds at more frequencies.

These features of driven harmonic oscillators apply to a huge variety of systems. When you tune a radio, for example, you are adjusting its resonant frequency so that it only oscillates to the desired station's broadcast (driving) frequency. The more selective the radio is in discriminating between stations, the smaller its damping. Magnetic resonance imaging (MRI) is a widely used medical diagnostic tool in which atomic nuclei (mostly hydrogen nuclei) are made to resonate by incoming radio waves (on the order of 100 MHz). A child on a swing is driven by a parent at the swing's natural frequency to achieve maximum amplitude. In all of these cases, the efficiency of energy transfer from the driving force into the oscillator is best at resonance. Speed bumps and gravel roads prove that even a car's suspension system is not immune to resonance. In spite of finely engineered shock absorbers, which ordinarily convert mechanical energy to thermal energy almost as fast as it comes in, speed bumps still cause a large-amplitude oscillation. On gravel roads that are corrugated, you may have noticed that if you travel at the "wrong" speed, the bumps are very noticeable whereas at other speeds you may hardly feel the bumps at all. [link] shows a photograph of a famous example (the Tacoma Narrows Bridge) of the destructive effects of a driven harmonic oscillation. The Millennium Bridge in London was closed for a short period of time for the same reason while inspections were carried out.

In our bodies, the chest cavity is a clear example of a system at resonance. The diaphragm and chest wall drive the oscillations of the chest cavity which result in the lungs inflating and deflating. The system is critically damped and the muscular diaphragm oscillates at the resonant value for the system, making it highly efficient.



In 1940, the Tacoma Narrows Bridge in Washington state collapsed. Heavy cross winds drove the bridge into oscillations at its resonant frequency. Damping decreased when support cables broke loose and started to slip over the towers, allowing increasingly greater amplitudes until the structure failed (credit: PRI's *Studio 360*, via Flickr)

Exercise:
Check Your Understanding

Problem:

A famous magic trick involves a performer singing a note toward a crystal glass until the glass shatters. Explain why the trick works in terms of resonance and natural frequency.

Solution:

The performer must be singing a note that corresponds to the natural frequency of the glass. As the sound wave is directed at the glass, the glass responds by resonating at the same frequency as the sound wave.

With enough energy introduced into the system, the glass begins to vibrate and eventually shatters.

Section Summary

- A system's natural frequency is the frequency at which the system will oscillate if not affected by driving or damping forces.
- A periodic force driving a harmonic oscillator at its natural frequency produces resonance. The system is said to resonate.
- The less damping a system has, the higher the amplitude of the forced oscillations near resonance. The more damping a system has, the broader response it has to varying driving frequencies.

Conceptual Questions

Exercise:

Problem:

Why are soldiers in general ordered to "route step" (walk out of step) across a bridge?

Problems & Exercises

Exercise:

Problem:

How much energy must the shock absorbers of a 1200-kg car dissipate in order to damp a bounce that initially has a velocity of 0.800 m/s at the equilibrium position? Assume the car returns to its original vertical position.

Solution:

384 J

Exercise:

Problem:

(a) How much will a spring that has a force constant of 40.0 N/m be stretched by an object with a mass of 0.500 kg when hung motionless from the spring? (b) Calculate the decrease in gravitational potential energy of the 0.500-kg object when it descends this distance. (c) Part of this gravitational energy goes into the spring. Calculate the energy stored in the spring by this stretch, and compare it with the gravitational potential energy. Explain where the rest of the energy might go.

Solution:

(a). 0.123 m

(b). −0.600 J

(c). 0.300 J. The rest of the energy may go into heat caused by friction and other damping forces.

Exercise:

Problem:

Engineering Application: A suspension bridge oscillates with an effective force constant of $1.00 \times 10^8\ \mathrm{N/m}$. (a) How much energy is needed to make it oscillate with an amplitude of 0.100 m? (b) If soldiers march across the bridge with a cadence equal to the bridge's natural frequency and impart $1.00 \times 10^4\ \mathrm{J}$ of energy each second, how long does it take for the bridge's oscillations to go from 0.100 m to 0.500 m amplitude?

Solution:

(a) $5.00 \times 10^5\ \mathrm{J}$

(b) $1.20{\times}10^3$ s

Glossary

natural frequency
: the frequency at which a system would oscillate if there were no driving and no damping forces

resonance
: the phenomenon of driving a system with a frequency equal to the system's natural frequency

resonate
: a system being driven at its natural frequency

9.9 - Derived copy of Waves

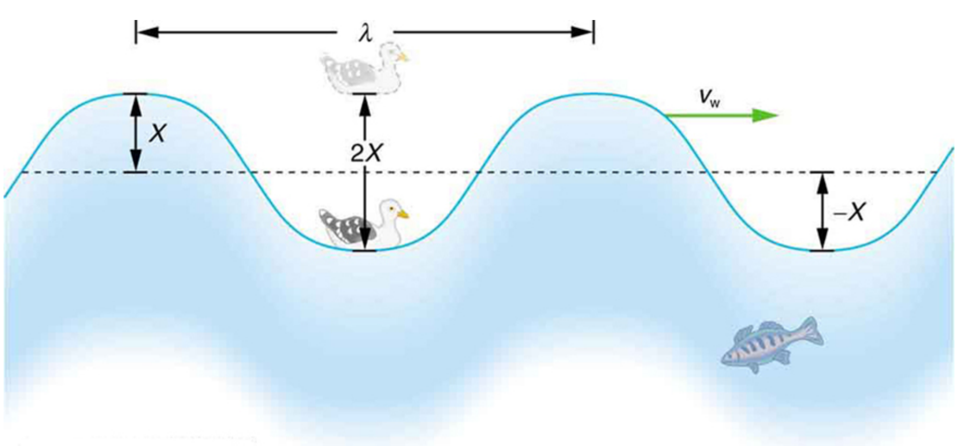
- State the characteristics of a wave.
- Calculate the velocity of wave propagation.

Waves in the ocean behave similarly to all other types of waves. (credit: Steve Jurveston, Flickr)

What do we mean when we say something is a wave? The most intuitive and easiest wave to imagine is the familiar water wave. More precisely, a **wave** is a disturbance that propagates, or moves from the place it was created. For water waves, the disturbance is in the surface of the water, perhaps created by a rock thrown into a pond or by a swimmer splashing the surface repeatedly. For sound waves, the disturbance is a change in air pressure, perhaps created by the oscillating cone inside a speaker. For earthquakes, there are several types of disturbances, including disturbance of Earth's surface and pressure disturbances under the surface. Even radio waves are most easily understood using an analogy with water waves. Visualizing water waves is useful because there is more to it than just a mental image. Water waves exhibit characteristics common to all waves, such as amplitude, period, frequency and energy. All wave characteristics can be described by a small set of underlying principles.

A wave is a disturbance that propagates, or moves from the place it was created. The simplest waves repeat themselves for several cycles and are associated with simple harmonic motion. Let us start by considering the simplified water wave in [link]. The wave is an up and down disturbance of the water surface. It causes a sea gull to move up and down in simple harmonic motion as the wave crests and troughs (peaks and valleys) pass under the bird. The time for one complete up and down motion is the wave's period T. The wave's frequency is $f = 1/T$, as usual. The wave itself moves to the right in the figure. This movement of the wave is actually the disturbance moving to the right, not the water itself (or the bird would move to the right). We define **wave velocity** v_w to be the speed at which the disturbance moves. Wave velocity is sometimes also called the *propagation velocity or propagation speed*, because the disturbance propagates from one location to another.

Note:
Misconception Alert
Many people think that water waves push water from one direction to another. In fact, the particles of water tend to stay in one location, save for moving up and down due to the energy in the wave. The energy moves forward through the water, but the water stays in one place. If you feel yourself pushed in an ocean, what you feel is the energy of the wave, not a rush of water.



An idealized ocean wave passes under a sea gull that bobs up and down in simple harmonic motion. The wave has a wavelength λ, which is the distance between adjacent identical parts of the wave. The up and down disturbance of the surface propagates parallel to the surface at a speed v_{w}.

The water wave in the figure also has a length associated with it, called its **wavelength** λ, the distance between adjacent identical parts of a wave. (λ is the distance parallel to the direction of propagation.) The speed of propagation v_{w} is the distance the wave travels in a given time, which is one wavelength in the time of one period. In equation form, that is

Equation:

$$v_{\text{w}} = \frac{\lambda}{T}$$

or

Equation:

$$v_{\text{w}} = f\lambda.$$

This fundamental relationship holds for all types of waves. For water waves, v_{w} is the speed of a surface wave; for sound, v_{w} is the speed of sound; and for visible light, v_{w} is the speed of light, for example.

Example:

Calculate the Velocity of Wave Propagation: Gull in the Ocean

Calculate the wave velocity of the ocean wave in [link] if the distance between wave crests is 10.0 m and the time for a sea gull to bob up and down is 5.00 s.

Strategy

We are asked to find v_w. The given information tells us that $\lambda = 10.0 \text{ m}$ and $T = 5.00 \text{ s}$. Therefore, we can use $v_w = \frac{\lambda}{T}$ to find the wave velocity.
Solution

1. Enter the known values into $v_w = \frac{\lambda}{T}$:
   Equation:

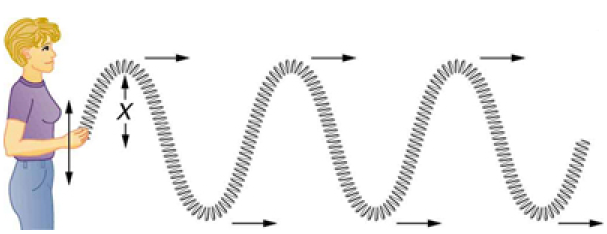
$$v_w = \frac{10.0 \text{ m}}{5.00 \text{ s}}.$$

2. Solve for v_w to find $v_w = 2.00 \text{ m/s}$.

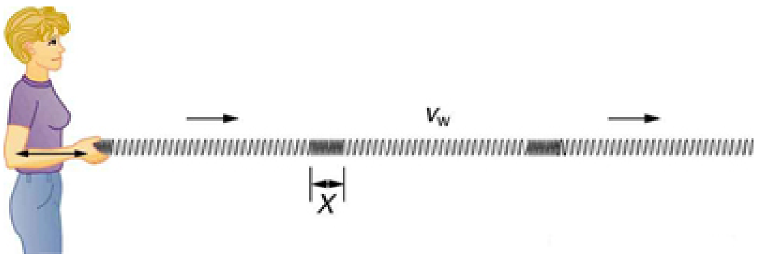
Discussion
This slow speed seems reasonable for an ocean wave. Note that the wave moves to the right in the figure at this speed, not the varying speed at which the sea gull moves up and down.

Transverse and Longitudinal Waves

A simple wave consists of a periodic disturbance that propagates from one place to another. The wave in [link] propagates in the horizontal direction while the surface is disturbed in the vertical direction. Such a wave is called a **transverse wave** or shear wave; in such a wave, the disturbance is perpendicular to the direction of propagation. In contrast, in a **longitudinal wave** or compressional wave, the disturbance is parallel to the direction of propagation. [link] shows an example of a longitudinal wave. The size of the disturbance is its amplitude X and is completely independent of the speed of propagation v_w.



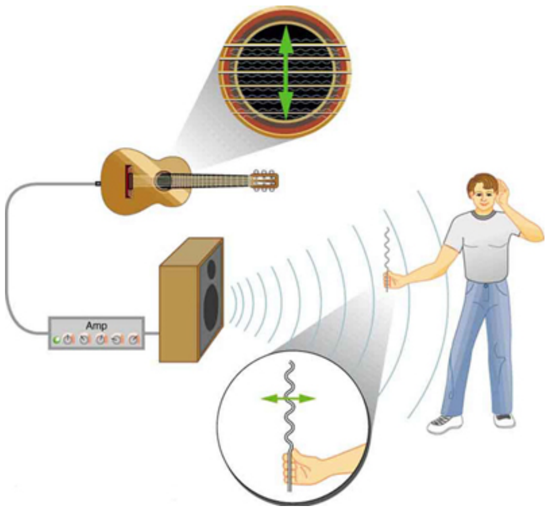
In this example of a transverse wave, the wave propagates horizontally, and the disturbance in the cord is in the vertical direction.



In this example of a longitudinal wave, the wave propagates horizontally, and the disturbance in the cord is also in the horizontal direction.

Waves may be transverse, longitudinal, or *a combination of the two*. (Water waves are actually a combination of transverse and longitudinal. The simplified water wave illustrated in [link] shows no longitudinal motion of the bird.) The waves on the strings of musical instruments are transverse—so are electromagnetic waves, such as visible light.

Sound waves in air and water are longitudinal. Their disturbances are periodic variations in pressure that are transmitted in fluids. Fluids do not have appreciable shear strength, and thus the sound waves in them must be longitudinal or compressional. Sound in solids can be both longitudinal and transverse.



The wave on a guitar string is transverse. The sound wave rattles a sheet of paper in a direction that shows the sound wave is longitudinal.

Earthquake waves under Earth’s surface also have both longitudinal and transverse components (called compressional or P-waves and shear or S-waves, respectively). These components have important individual characteristics—they propagate at different speeds, for example. Earthquakes also have surface waves that are similar to surface waves on water.

Exercise:

Check Your Understanding

Problem:

Why is it important to differentiate between longitudinal and transverse waves?

Solution:

In the different types of waves, energy can propagate in a different direction relative to the motion of the wave. This is important to

understand how different types of waves affect the materials around them.

Section Summary

- A wave is a disturbance that moves from the point of creation with a wave velocity v_{w}.
- A wave has a wavelength λ, which is the distance between adjacent identical parts of the wave.
- Wave velocity and wavelength are related to the wave's frequency and period by $v_{\text{w}} = \frac{\lambda}{T}$ or $v_{\text{w}} = f\lambda$.
- A transverse wave has a disturbance perpendicular to its direction of propagation, whereas a longitudinal wave has a disturbance parallel to its direction of propagation.

Problems & Exercises

Exercise:

Problem:

Storms in the South Pacific can create waves that travel all the way to the California coast, which are 12,000 km away. How long does it take them if they travel at 15.0 m/s?

Solution:
Equation:

$$t = 9.26\text{ d}$$

Exercise:

Problem:

Wind gusts create ripples on the ocean that have a wavelength of 5.00 cm and propagate at 2.00 m/s. What is their frequency?

Solution:
Equation:

$$f = 40.0 \text{ Hz}$$

Exercise:

Problem:

How many times a minute does a boat bob up and down on ocean waves that have a wavelength of 40.0 m and a propagation speed of 5.00 m/s?

Exercise:

Problem:

What is the wavelength of an earthquake that shakes you with a frequency of 10.0 Hz and gets to another city 84.0 km away in 12.0 s?

Solution:
Equation:

$$\lambda = 700 \text{ m}$$

Exercise:

Problem:

Your ear is capable of differentiating sounds that arrive at the ear just 1.00 ms apart. What is the minimum distance between two speakers that produce sounds that arrive at noticeably different times on a day when the speed of sound is 340 m/s?

Solution:
Equation:

$$d = 34.0 \text{ cm}$$

Glossary

longitudinal wave
: a wave in which the disturbance is parallel to the direction of propagation

transverse wave
: a wave in which the disturbance is perpendicular to the direction of propagation

wave velocity
: the speed at which the disturbance moves. Also called the propagation velocity or propagation speed

wavelength
: the distance between adjacent identical parts of a wave

9.10 - Superposition and Interference

- Explain standing waves.
- Describe the mathematical representation of overtones and beat frequency.

These waves result from the superposition of several waves from different sources, producing a complex pattern. (credit: waterborough, Wikimedia Commons)

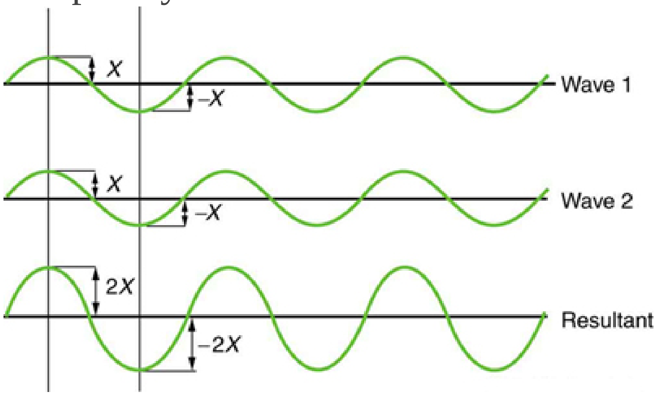
Most waves do not look very simple. They look more like the waves in [link] than like the simple water wave considered in Waves. (Simple waves may be created by a simple harmonic oscillation, and thus have a sinusoidal shape). Complex waves are more interesting, even beautiful, but they look formidable. Most waves appear complex because they result from several simple waves adding together. Luckily, the rules for adding waves are quite simple.

When two or more waves arrive at the same point, they superimpose themselves on one another. More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called **superposition**. Each disturbance corresponds to a force, and forces add. If the disturbances are along the same line, then the resulting wave is a simple

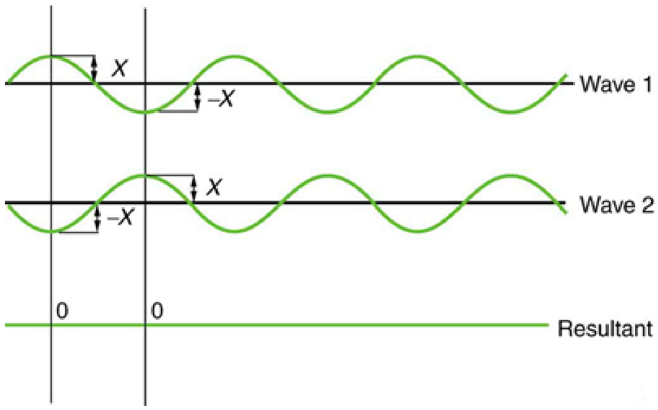
addition of the disturbances of the individual waves—that is, their amplitudes add. [link] and [link] illustrate superposition in two special cases, both of which produce simple results.

[link] shows two identical waves that arrive at the same point exactly in phase. The crests of the two waves are precisely aligned, as are the troughs. This superposition produces pure **constructive interference**. Because the disturbances add, pure constructive interference produces a wave that has twice the amplitude of the individual waves, but has the same wavelength.

[link] shows two identical waves that arrive exactly out of phase—that is, precisely aligned crest to trough—producing pure **destructive interference**. Because the disturbances are in the opposite direction for this superposition, the resulting amplitude is zero for pure destructive interference—the waves completely cancel.



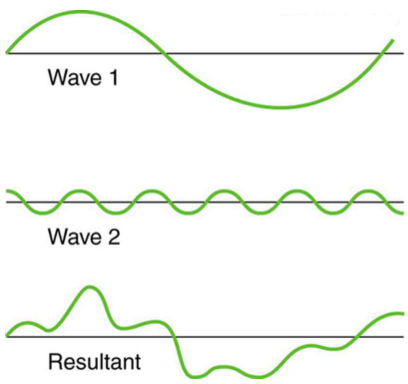
Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength.



Pure destructive interference of two identical waves produces zero amplitude, or complete cancellation.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference and can vary from place to place and time to time. Sound from a stereo, for example, can be loud in one spot and quiet in another. Varying loudness means the sound waves add partially constructively and partially destructively at different locations. A stereo has at least two speakers creating sound waves, and waves can reflect from walls. All these waves superimpose. An example of sounds that vary over time from constructive to destructive is found in the combined whine of airplane jets heard by a stationary passenger. The combined sound can fluctuate up and down in volume as the sound from the two engines varies in time from constructive to destructive. These examples are of waves that are similar.

An example of the superposition of two dissimilar waves is shown in [link]. Here again, the disturbances add and subtract, producing a more complicated looking wave.

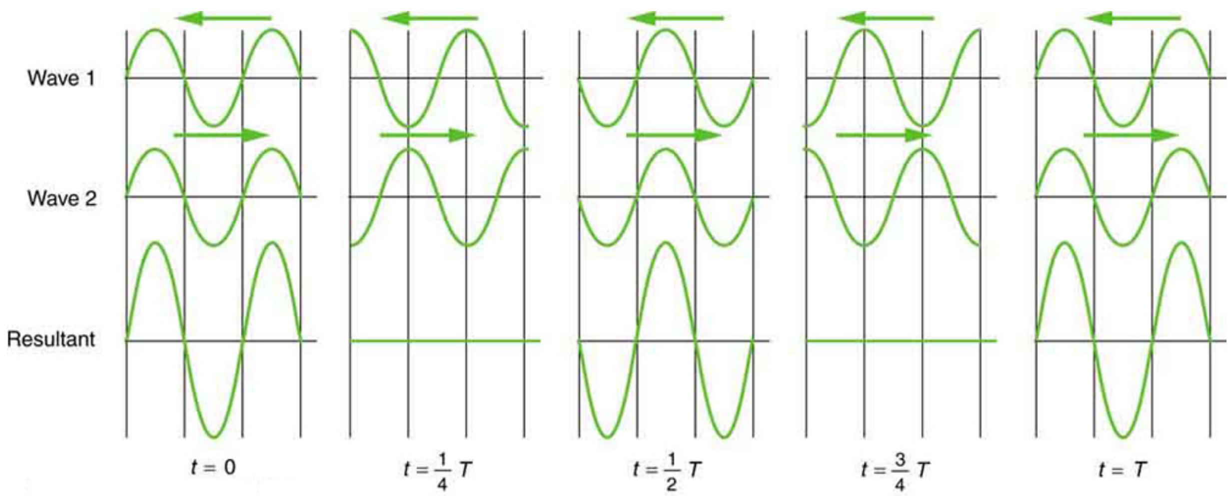


Superposition of non-identical waves exhibits both constructive and destructive interference.

Standing Waves

Sometimes waves do not seem to move; rather, they just vibrate in place. Unmoving waves can be seen on the surface of a glass of milk in a refrigerator, for example. Vibrations from the refrigerator motor create waves on the milk that oscillate up and down but do not seem to move across the surface. These waves are formed by the superposition of two or more moving waves, such as illustrated in [link] for two identical waves moving in opposite directions. The waves move through each other with their disturbances adding as they go by. If the two waves have the same amplitude and wavelength, then they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a **standing wave**. Waves on the glass of milk are one example of standing waves. There are other standing waves, such as on guitar strings and in organ pipes. With the glass of milk, the two waves that produce standing waves may come from reflections from the side of the glass.

A closer look at earthquakes provides evidence for conditions appropriate for resonance, standing waves, and constructive and destructive interference. A building may be vibrated for several seconds with a driving frequency matching that of the natural frequency of vibration of the building—producing a resonance resulting in one building collapsing while neighboring buildings do not. Often buildings of a certain height are devastated while other taller buildings remain intact. The building height matches the condition for setting up a standing wave for that particular height. As the earthquake waves travel along the surface of Earth and reflect off denser rocks, constructive interference occurs at certain points. Often areas closer to the epicenter are not damaged while areas farther away are damaged.

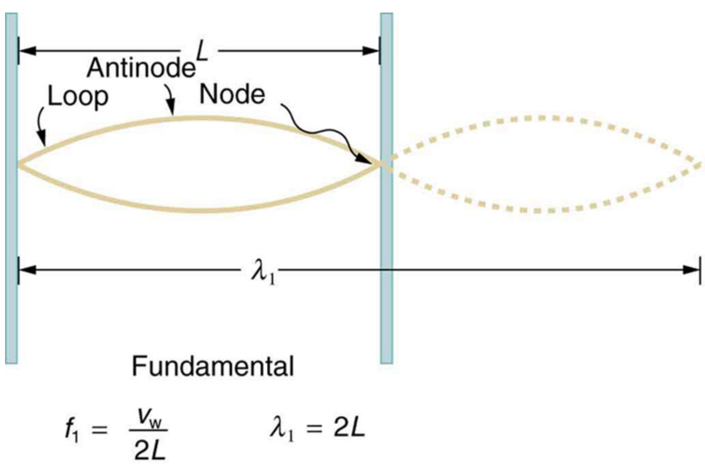


Standing wave created by the superposition of two identical waves moving in opposite directions. The oscillations are at fixed locations in space and result from alternately constructive and destructive interference.

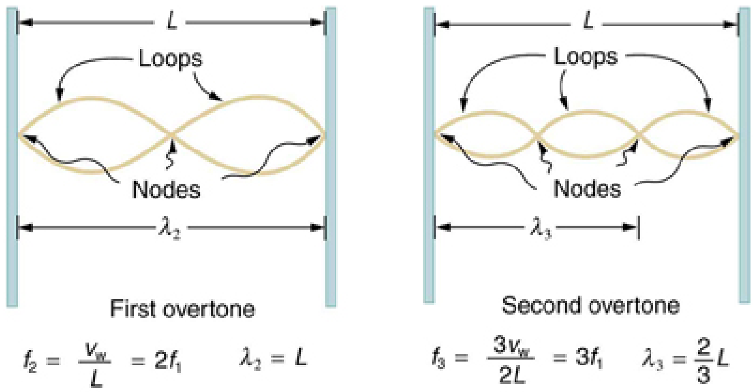
Standing waves are also found on the strings of musical instruments and are due to reflections of waves from the ends of the string. [link] and [link] show three standing waves that can be created on a string that is fixed at both ends. **Nodes** are the points where the string does not move; more

generally, nodes are where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, too, because the string cannot move there. The word **antinode** is used to denote the location of maximum amplitude in standing waves. Standing waves on strings have a frequency that is related to the propagation speed v_w of the disturbance on the string. The wavelength λ is determined by the distance between the points where the string is fixed in place.

The lowest frequency, called the **fundamental frequency**, is thus for the longest wavelength, which is seen to be $\lambda_1 = 2L$. Therefore, the fundamental frequency is $f_1 = v_w/\lambda_1 = v_w/2L$. In this case, the **overtones** or harmonics are multiples of the fundamental frequency. As seen in [link], the first harmonic can easily be calculated since $\lambda_2 = L$. Thus, $f_2 = v_w/\lambda_2 = v_w/2L = 2f_1$. Similarly, $f_3 = 3f_1$, and so on. All of these frequencies can be changed by adjusting the tension in the string. The greater the tension, the greater v_w is and the higher the frequencies. This observation is familiar to anyone who has ever observed a string instrument being tuned. We will see in later chapters that standing waves are crucial to many resonance phenomena, such as in sounding boxes on string instruments.



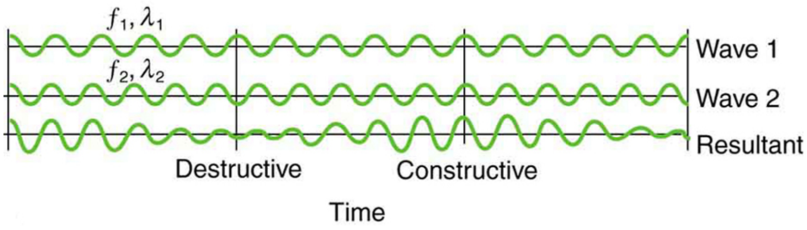
The figure shows a string oscillating at its fundamental frequency.



First and second harmonic frequencies are shown.

Beats

Striking two adjacent keys on a piano produces a warbling combination usually considered to be unpleasant. The superposition of two waves of similar but not identical frequencies is the culprit. Another example is often noticeable in jet aircraft, particularly the two-engine variety, while taxiing. The combined sound of the engines goes up and down in loudness. This varying loudness happens because the sound waves have similar but not identical frequencies. The discordant warbling of the piano and the fluctuating loudness of the jet engine noise are both due to alternately constructive and destructive interference as the two waves go in and out of phase. [link] illustrates this graphically.



Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes. The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

The wave resulting from the superposition of two similar-frequency waves has a frequency that is the average of the two. This wave fluctuates in amplitude, or *beats*, with a frequency called the **beat frequency**. We can determine the beat frequency by adding two waves together mathematically. Note that a wave can be represented at one point in space as

Equation:

$$x = X\cos\left(\frac{2\pi\, t}{T}\right) = X\cos(2\pi\ \mathrm{ft}),$$

where $f = 1/T$ is the frequency of the wave. Adding two waves that have different frequencies but identical amplitudes produces a resultant

Equation:

$$x = x_1 + x_2.$$

More specifically,

Equation:

$$x = X\cos(2\pi\ f_1 t) + X\cos(2\pi\ f_2 t).$$

Using a trigonometric identity, it can be shown that
Equation:

$$x=2X\cos(\pi f_{\text{B}}t)\cos(2\pi f_{\text{ave}}t),$$

where
Equation:

$$f_{\text{B}}=\mid f_1-f_2\mid$$

is the beat frequency, and f_{ave} is the average of f_1 and f_2. These results mean that the resultant wave has twice the amplitude and the average frequency of the two superimposed waves, but it also fluctuates in overall amplitude at the beat frequency f_{B}. The first cosine term in the expression effectively causes the amplitude to go up and down. The second cosine term is the wave with frequency f_{ave}. This result is valid for all types of waves. However, if it is a sound wave, providing the two frequencies are similar, then what we hear is an average frequency that gets louder and softer (or warbles) at the beat frequency.

Note:
Making Career Connections
Piano tuners use beats routinely in their work. When comparing a note with a tuning fork, they listen for beats and adjust the string until the beats go away (to zero frequency). For example, if the tuning fork has a 256 Hz frequency and two beats per second are heard, then the other frequency is either 254 or 258 Hz. Most keys hit multiple strings, and these strings are actually adjusted until they have nearly the same frequency and give a slow beat for richness. Twelve-string guitars and mandolins are also tuned using beats.

While beats may sometimes be annoying in audible sounds, we will find that beats have many applications. Observing beats is a very useful way to compare similar frequencies. There are applications of beats as apparently disparate as in ultrasonic imaging and radar speed traps.

Exercise:

Check Your Understanding

Problem:

Imagine you are holding one end of a jump rope, and your friend holds the other. If your friend holds her end still, you can move your end up and down, creating a transverse wave. If your friend then begins to move her end up and down, generating a wave in the opposite direction, what resultant wave forms would you expect to see in the jump rope?

Solution:

The rope would alternate between having waves with amplitudes two times the original amplitude and reaching equilibrium with no amplitude at all. The wavelengths will result in both constructive and destructive interference

Exercise:

Check Your Understanding

Problem: Define nodes and antinodes.

Solution:

Nodes are areas of wave interference where there is no motion. Antinodes are areas of wave interference where the motion is at its maximum point.

Exercise:

Check Your Understanding

Problem:

You hook up a stereo system. When you test the system, you notice that in one corner of the room, the sounds seem dull. In another area, the sounds seem excessively loud. Describe how the sound moving about the room could result in these effects.

Solution:

With multiple speakers putting out sounds into the room, and these sounds bouncing off walls, there is bound to be some wave interference. In the dull areas, the interference is probably mostly destructive. In the louder areas, the interference is probably mostly constructive.

Section Summary

- Superposition is the combination of two waves at the same location.
- Constructive interference occurs when two identical waves are superimposed in phase.
- Destructive interference occurs when two identical waves are superimposed exactly out of phase.
- A standing wave is one in which two waves superimpose to produce a wave that varies in amplitude but does not propagate.
- Nodes are points of no motion in standing waves.
- An antinode is the location of maximum amplitude of a standing wave.
- Waves on a string are resonant standing waves with a fundamental frequency and can occur at higher multiples of the fundamental, called overtones or harmonics.
- Beats occur when waves of similar frequencies f_1 and f_2 are superimposed. The resulting amplitude oscillates with a beat frequency given by
  Equation:
  $$f_{\mathrm{B}} = \mid f_1 - f_2 \mid .$$

Conceptual Questions

Exercise:

Problem:

Speakers in stereo systems have two color-coded terminals to indicate how to hook up the wires. If the wires are reversed, the speaker moves in a direction opposite that of a properly connected speaker. Explain why it is important to have both speakers connected the same way.

Problems & Exercises

Exercise:

Problem:

A car has two horns, one emitting a frequency of 199 Hz and the other emitting a frequency of 203 Hz. What beat frequency do they produce?

Solution:

$f = 4\text{ Hz}$

Exercise:

Problem:

A wave traveling on a Slinky® that is stretched to 4 m takes 2.4 s to travel the length of the Slinky and back again. (a) What is the speed of the wave? (b) Using the same Slinky stretched to the same length, a standing wave is created which consists of three antinodes and four nodes. At what frequency must the Slinky be oscillating?

Solution:

(a) 3.33 m/s

(b) 1.25 Hz

Exercise:

Problem:

Three adjacent keys on a piano (F, F-sharp, and G) are struck simultaneously, producing frequencies of 349, 370, and 392 Hz. What beat frequencies are produced by this discordant combination?

Glossary

antinode
: the location of maximum amplitude in standing waves

beat frequency
: the frequency of the amplitude fluctuations of a wave

constructive interference
: when two waves arrive at the same point exactly in phase; that is, the crests of the two waves are precisely aligned, as are the troughs

destructive interference
: when two identical waves arrive at the same point exactly out of phase; that is, precisely aligned crest to trough

fundamental frequency
: the lowest frequency of a periodic waveform

nodes
: the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave

overtones
: multiples of the fundamental frequency of a sound

superposition
: the phenomenon that occurs when two or more waves arrive at the same point

9.11 - Energy in Waves: Intensity

- Calculate the intensity and the power of rays and waves.

The destructive effect of an earthquake is palpable evidence of the energy carried in these waves. The Richter scale rating of earthquakes is related to both their amplitude and the energy they carry. (credit: Petty Officer 2nd Class Candice Villarreal, U.S. Navy)

All waves carry energy. The energy of some waves can be directly observed. Earthquakes can shake whole cities to the ground, performing the work of thousands of wrecking balls.

Loud sounds pulverize nerve cells in the inner ear, causing permanent hearing loss. Ultrasound is used for deep-heat treatment of muscle strains. A laser beam can burn away a malignancy. Water waves chew up beaches.

The amount of energy in a wave is related to its amplitude. Large-amplitude earthquakes produce large ground displacements. Loud sounds have higher pressure amplitudes and come from larger-amplitude source vibrations than

soft sounds. Large ocean breakers churn up the shore more than small ones. More quantitatively, a wave is a displacement that is resisted by a restoring force. The larger the displacement x, the larger the force $F = \mathrm{kx}$ needed to create it. Because work W is related to force multiplied by distance (Fx) and energy is put into the wave by the work done to create it, the energy in a wave is related to amplitude. In fact, a wave's energy is directly proportional to its amplitude squared because

Equation:

$$W \propto \mathrm{Fx} = \mathrm{kx}^2.$$

The energy effects of a wave depend on time as well as amplitude. For example, the longer deep-heat ultrasound is applied, the more energy it transfers. Waves can also be concentrated or spread out. Sunlight, for example, can be focused to burn wood. Earthquakes spread out, so they do less damage the farther they get from the source. In both cases, changing the area the waves cover has important effects. All these pertinent factors are included in the definition of **intensity** I as power per unit area:

Equation:

$$I = \frac{P}{A}$$

where P is the power carried by the wave through area A. The definition of intensity is valid for any energy in transit, including that carried by waves. The SI unit for intensity is watts per square meter ($\mathrm{W/m^2}$). For example, infrared and visible energy from the Sun impinge on Earth at an intensity of $1300\ \mathrm{W/m^2}$ just above the atmosphere. There are other intensity-related units in use, too. The most common is the decibel. For example, a 90 decibel sound level corresponds to an intensity of $10^{-3}\ \mathrm{W/m^2}$. (This quantity is not much power per unit area considering that 90 decibels is a relatively high sound level. Decibels will be discussed in some detail in a later chapter.

Example:
Calculating intensity and power: How much energy is in a ray of sunlight?

The average intensity of sunlight on Earth's surface is about $700\ \mathrm{W/m^2}$.
(a) Calculate the amount of energy that falls on a solar collector having an area of $0.500\ \mathrm{m}^2$ in 4.00 h.
(b) What intensity would such sunlight have if concentrated by a magnifying glass onto an area 200 times smaller than its own?

Strategy a

Because power is energy per unit time or $P=\frac{E}{t}$, the definition of intensity can be written as $I=\frac{P}{A}=\frac{E/t}{A}$, and this equation can be solved for E with the given information.

Solution a

1. Begin with the equation that states the definition of intensity:
   Equation:
   $$I=\frac{P}{A}.$$
2. Replace P with its equivalent E/t:
   Equation:
   $$I=\frac{E/t}{A}.$$
3. Solve for E:
   Equation:
   $$E=\mathrm{IAt}.$$
4. Substitute known values into the equation:
   Equation:
   $$E=\left(700\ \mathrm{W/m^2}\right)\left(0.500\ \mathrm{m^2}\right)\left[(4.00\ \mathrm{h})(3600\ \mathrm{s/h})\right].$$
5. Calculate to find E and convert units:

Equation:

$$5.04 \times 10^6 \text{ J},$$

Discussion a
The energy falling on the solar collector in 4 h in part is enough to be useful—for example, for heating a significant amount of water.
Strategy b
Taking a ratio of new intensity to old intensity and using primes for the new quantities, we will find that it depends on the ratio of the areas. All other quantities will cancel.
Solution b

1. Take the ratio of intensities, which yields:
   Equation:

$$\frac{I\prime}{I} = \frac{P\prime/A\prime}{P/A} = \frac{A}{A\prime}\left(\text{The powers cancel because } P\prime = P\right).$$

2. Identify the knowns:
   Equation:

$$A = 200A\prime,$$

   Equation:

$$\frac{I\prime}{I} = 200.$$

3. Substitute known quantities:
   Equation:

$$I\prime = 200I = 200\left(700 \text{ W/m}^2\right).$$

4. Calculate to find $I\prime$:
   Equation:

$$I\prime = 1.40 \times 10^5\ \text{W/m}^2.$$

Discussion b

Decreasing the area increases the intensity considerably. The intensity of the concentrated sunlight could even start a fire.

Example:

Determine the combined intensity of two waves: Perfect constructive interference

If two identical waves, each having an intensity of $1.00\ \text{W/m}^2$, interfere perfectly constructively, what is the intensity of the resulting wave?

Strategy

We know from Superposition and Interference that when two identical waves, which have equal amplitudes X, interfere perfectly constructively, the resulting wave has an amplitude of $2X$. Because a wave's intensity is proportional to amplitude squared, the intensity of the resulting wave is four times as great as in the individual waves.

Solution

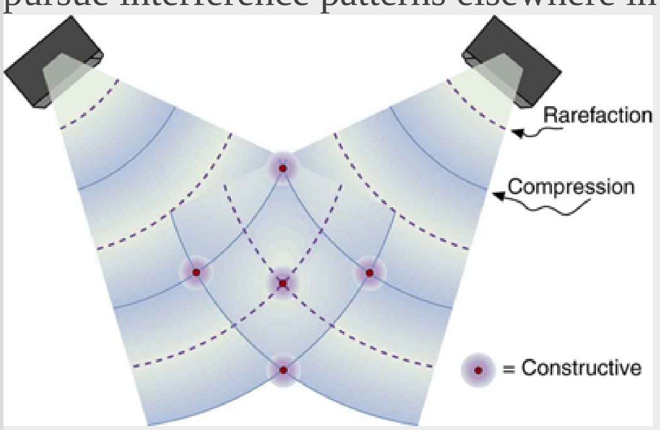
1. Recall that intensity is proportional to amplitude squared.
2. Calculate the new amplitude:
   Equation:
   $$I\prime \propto (X\prime)^2 = (2X)^2 = 4X^2.$$
3. Recall that the intensity of the old amplitude was:
   Equation:
   $$I \propto X^2.$$
4. Take the ratio of new intensity to the old intensity. This gives:
   Equation:
   $$\frac{I\prime}{I} = 4.$$

5. Calculate to find $I\prime$:
   Equation:

$$I\prime = 4I = 4.00\ \mathrm{W/m^2}.$$

Discussion

The intensity goes up by a factor of 4 when the amplitude doubles. This answer is a little disquieting. The two individual waves each have intensities of $1.00\ \mathrm{W/m^2}$, yet their sum has an intensity of $4.00\ \mathrm{W/m^2}$, which may appear to violate conservation of energy. This violation, of course, cannot happen. What does happen is intriguing. The area over which the intensity is $4.00\ \mathrm{W/m^2}$ is much less than the area covered by the two waves before they interfered. There are other areas where the intensity is zero. The addition of waves is not as simple as our first look in Superposition and Interference suggested. We actually get a pattern of both constructive interference and destructive interference whenever two waves are added. For example, if we have two stereo speakers putting out $1.00\ \mathrm{W/m^2}$ each, there will be places in the room where the intensity is $4.00\ \mathrm{W/m^2}$, other places where the intensity is zero, and others in between. [link] shows what this interference might look like. We will pursue interference patterns elsewhere in this text.



These stereo speakers produce both constructive interference and destructive interference in the room, a property common to the

superposition of all types of waves. The shading is proportional to intensity.

Exercise:
Check Your Understanding

Problem:

Which measurement of a wave is most important when determining the wave's intensity?

Solution:

Amplitude, because a wave’s energy is directly proportional to its amplitude squared.

Section Summary

Intensity is defined to be the power per unit area:

$I=\frac{P}{A}$ and has units of $\mathrm{W/m^2}$.

Conceptual Questions

Exercise:

Problem:

Two identical waves undergo pure constructive interference. Is the resultant intensity twice that of the individual waves? Explain your answer.

Exercise:

Problem:

Circular water waves decrease in amplitude as they move away from where a rock is dropped. Explain why.

Glossary

intensity
: power per unit area